Maritime University of Szczecin

Faculty of Marine Engineering

Department of Physics and Chemistry

Physics Laboratory

Laboratory Manual

Confirming Steiner's theorem

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Equipment:

1. A device for testing the rotary motion possessing three mandrels for winding the rope. Mandrels' diameters: d_1 = 23.4 mm, d_2 = 33.6 mm, d_3 = 45.6 mm.
2. Two types of weights for the rotating disc.
3. A set of weights with hooks.
4. Rope.
5. Stopwatch.
6. Caliper.

Attention! The device for testing the rotary motion is very delicate. Exercise must be performed with caution-so as not to damage the device. In particular, the disc should be stopped by gentle grasping the base or the top of the reel.

Exercise:

I. Calculation of the moment of inertia from the definition

1. Check the mass and measure geometrical dimensions of the smaller weight.
2. Calculate density of the smaller weight for the rotating disc:

$$\rho_S = \frac{m_S}{V_S} = \frac{m_S}{\pi r_{1S}^2 h_{1S} + \pi r_{2S}^2 (h_{2S} - h_{1S})}.$$

3. Calculate the moment of inertia of the smaller weight for the rotating disc:

$$I_{0S} = \frac{1}{2}\pi\rho_S\left(r_{1S}^4 h_{1S} + r_{2S}^4 (h_{2S} - h_{1S})\right)$$

4. Repeat steps described in 1.1 – 1.3 for the larger weight for the rotating disc.

II. Determination of the moment o inertia of the disc without weights.

1. Determine mass *m* of the weight.
2. Tighten the weight to one of the ends of the rope. Insert the rope through the pulley and hook the other end to the holder located next to the drum (mandrel) indicated by the teacher.
3. Wind the rope on the drum. Make sure that the rope coils forming a single layer and does not touch the table below the pulley. Position the disk in a way that the mark placed on its edge is in front of the pulley.
4. Release the disc, at the same time turning on the stopwatch. Measure time *t* of four full disc rotations.
5. Repeat steps described in 2 – 4 twice.
6. Calculate mean value $\bar{t}$ of time of the disc rotation. Calculate the angular acceleration of the disc:

$$\varepsilon = \frac{2\alpha}{\bar{t}^2}$$

7. Calculate the torque M_N of the rope tension:

$$M_N = m(g - \varepsilon r)r$$

8. From the 2nd principle of dynamics for rotary movement

$$M_N = I_0 \varepsilon$$

determine the moment of inertia I_0 of the disk without weights.

III. Determination of the moment of inertia of the disc with weights.

1. Place k smaller weights in the holes in the disk marked with the number 1 (according to the teacher's instructions). Repeat steps described in II.2 – II.7.
2. Repeat the steps carried out in the previous point placing the weights in subsequent holes numbered $2 \leq n \leq 8$.
3. Present on a graph the dependence of $\frac{M_N}{\varepsilon}$ ratio from the square of the distance of weights from the center of the disc r_{ob}^2. Using the linear regression method, find the moment of inertia of the disk without weights and moment of inertia of weights $I_0 + kI_{0S}$, and the total mass of the weights km_m:

$$\frac{M_N}{\varepsilon} = I_0 + kI_{0S} + km_S \cdot {r_{ob}}^2$$

$$y \;=\; b \;+\; a \cdot x$$

Compare the results with the values obtained according to calculations described in points I and II.

4. Calculate the moment of inertia of the smaller weight in relation to the axis of rotation of the disk

$$I_S = I_{0S} + m_S {r_{ob}}^2.$$

Perform these calculations for all eight positions of the weights.

5. Calculate the resultant moment of inertia of the disk with weights

$$I = I_0 + kI_S$$

For all eight position of the weights.

6. Repeat steps described in III.1 – III.5 for bigger weights.
7. Formulate conclusions.

hole number	distance from the center of the disc [mm]
1	**146,8**
2	**102,1**
3	**122,2**
4	**69,6**
5	**161,7**
6	**91,8**
7	**125,7**
8	**51,1**

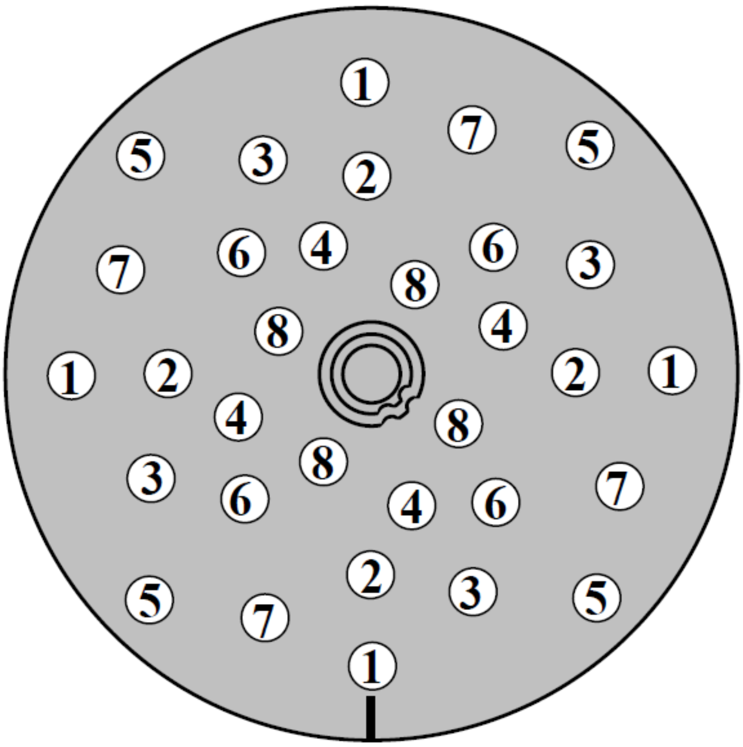
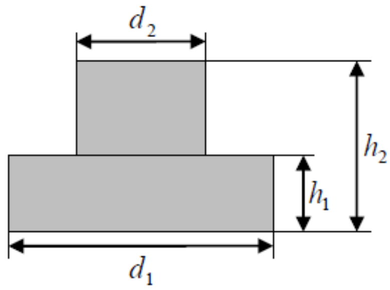


Table:



Smaller weight								
m_S [kg]	h_{1S} [m]	h_{2S} [m]	d_{1S} [m]	d_{2S} [m]	r_{1S} [m]	r_{2S} [m]	ρ_S [kg/m^3]	I_{0S} [kg·m^2]

Bigger weight								
m_B [kg]	h_{1B} [m]	h_{2B} [m]	d_{1B} [m]	d_{2B} [m]	r_{1B} [m]	r_{2B} [m]	ρ_B [kg/m^3]	I_{0B} [kg·m^2]

Weight's mass: m = ……..kg
Mandrel's radius: r = …….m

Rotation angle	**Time of rotation**	**Mean time**	**Angular acceleration**	**Torque of the rope tension**	**Moment of inertia of the disk without weights**
α [rad]	t_i [s]	$\bar{t}$ [s]	ε [rad/s^2]	M_N [N·m]	I_0 [kg·m^2]

weight’s mass: m = ……..kg ; mandrel’s radius: r = …….m ; number of weights: k = ………

Smaller weights												
Hole number	**Distance of the weight from the center of the disc**	**Angle of rotation**	**Time of rotation**	**Mean time**	**Angular acceleration**	**Torque of the rope tension**	**Square distance of the weight from the center of the disc**		**Moment of inertia of the disk without weights**	**Moment of inertia of the weight in relation to its axis of rotation**	**Moment of inertia of the weight in relation to the axis of rotation of the disk**	**Moment of inertia of the disk with weights**
n	r_{ob} [m]	α [rad]	t_i [s]	$\bar{t}$ [s]	ε [rad/s^2]	M_N [N·m]	r_{ob}^2 [m^2]	$\frac{M_N}{\varepsilon}$ [kg·m^2]	I_0 [kg·m^2]	I_{0S} [kg·m^2]	I_{Si} [kg·m^2]	I [kg·m^2]
1												
2												
3												
4												
5												
6												
7												
8												

a = …..…±…… b = ……..±……..

km_S = $I_0 + kI_{0S}$ =

weight's mass: m = ……..kg ; mandrel's radius: r = …….m ; number of weights: k = ………

Bigger weights												
Hole number	**Distance of the weight from the center of the disc**	**Angle of rotation**	**Time of rotation**	**Mean time**	**Angular acceleration**	**Torque of the rope tension**	**Square distance of the weight from the center of the disc**		**Moment of inertia of the disk without weights**	**Moment of inertia of the weight in relation to its axis of rotation**	**Moment of inertia of the weight in relation to the axis of rotation of the disk**	**Moment of inertia of the disk with weights**
n	r_{ob} [m]	α [rad]	t_i [s]	$\bar{t}$ [s]	ε [rad/s^2]	M_N [N·m]	r_{ob}^2 [m^2]	$\frac{M_N}{\varepsilon}$ [kg·m^2]	I_0 [kg·m^2]	I_{0B} [kg·m^2]	I_{Bi} [kg·m^2]	I [kg·m^2]
1												
2												
3												
4												
5												
6												
7												
8												

a = …..…±…… b = ……..±……..

km_B = $I_0 + kI_{0B}$ =