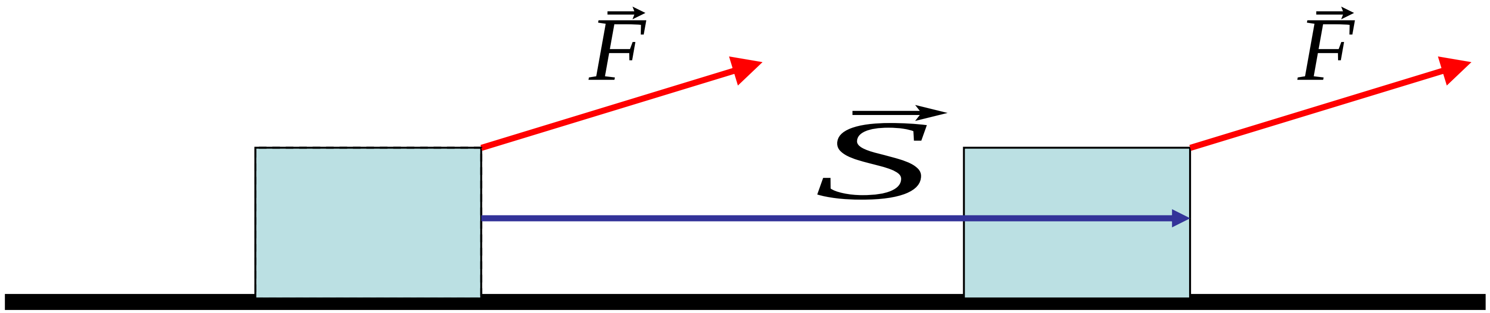
WORK

The work is done by a force acting on a body

while it undergoes a displacement.

$$W = FS$$

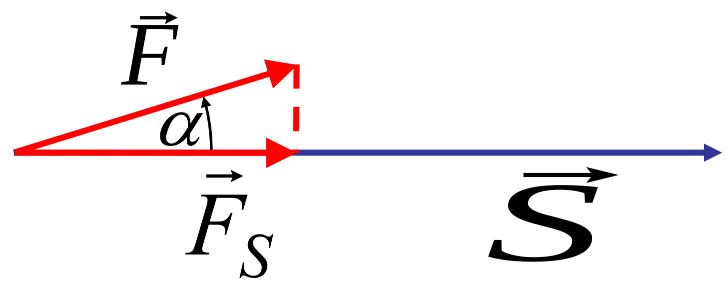
$$W = \vec{F} \cdot \vec{S}$$



$$W = \vec{F} \cdot \vec{S}$$

$$W_F = FS\cos(\alpha) = F_S S$$

$$\cos(\alpha) = \frac{F_S}{F}$$



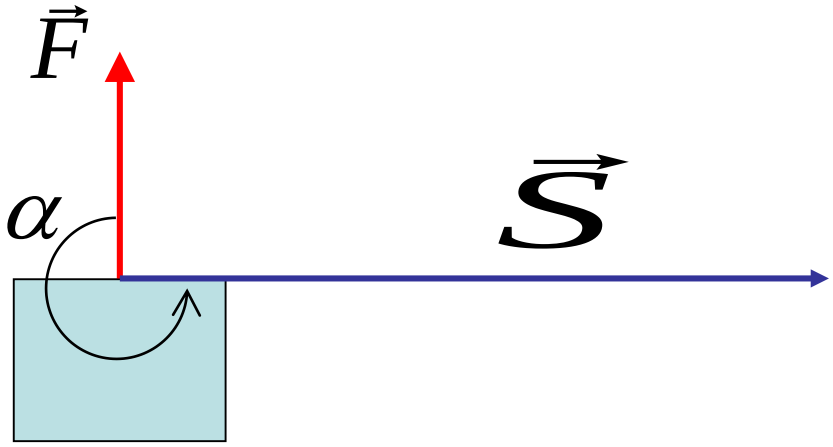
Work is done by a component of $\vec{F}$,

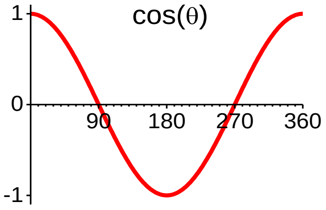
parallel to the displacement.

The work value depnds on:

- force $\vec{F}$ value
- displacement $\vec{S}$ *value*
- angle α between $\vec{F}$ and $\vec{S}$

Work done in horizontal shifting



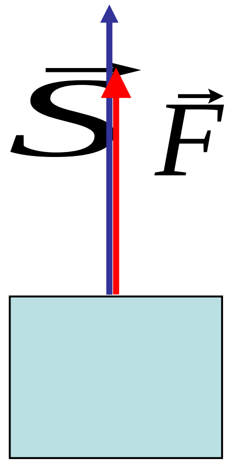


$$\alpha = 270^\circ \Rightarrow \cos(\alpha) = 0 \Rightarrow W_F = 0$$

The is no work in horizontal shiftng!!!

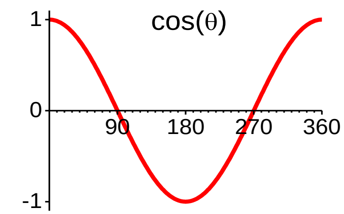
Carrying weights we do not work!!!

Work done in lifting an object



$\vec{S}$

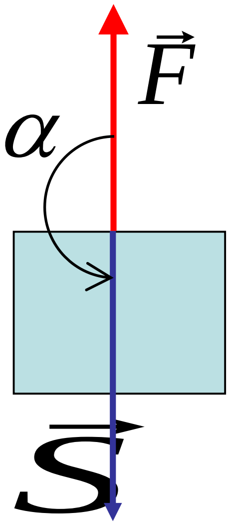
$$\vec{F} = -\vec{Q} = -m\vec{g}$$

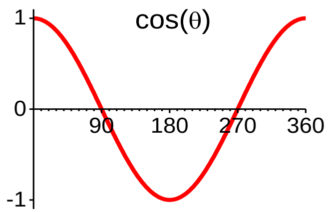


$$\alpha = 0^\circ \Rightarrow \cos(\alpha) = 1 \Rightarrow W_F = FS$$

When lifting, we perform positive work!!!

Work done in lowering an object



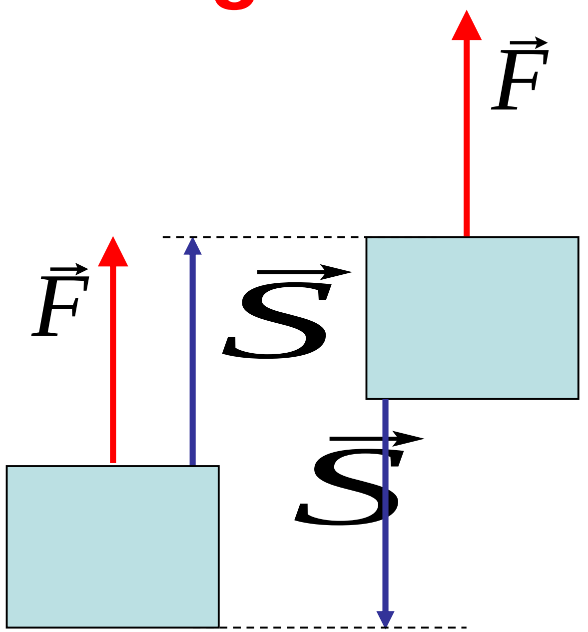


$$\alpha = 180° \Rightarrow \cos(\alpha) = -1 \Rightarrow W_F = -FS$$

When lowering, we perform negative work!!!

Lifting and lowering

$$\vec{F}_w = 0 \Rightarrow F = Q$$



$$W_{F_1} = FS = mgS$$

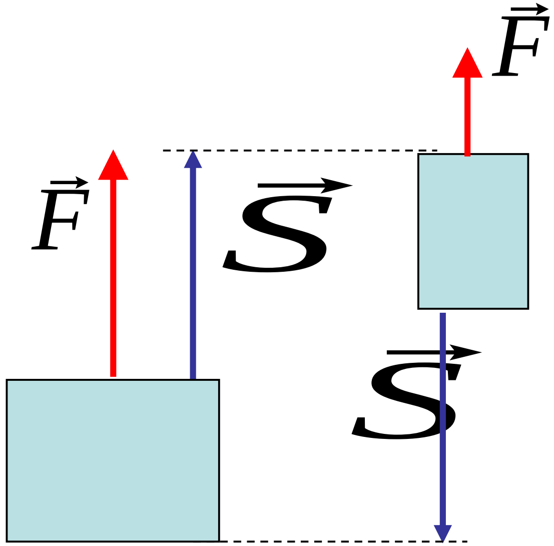
$$W_{F_2} = -FS = -mgS$$

$$W_F = W_{F_1} + W_{F_2} = 0$$

When lifting and lowering, the total work is zero!!!

Lifting and lowering with varying mass

$$\vec{F}_w = 0 \Rightarrow F = Q$$



$$W_{F_1} = FS = mgS$$

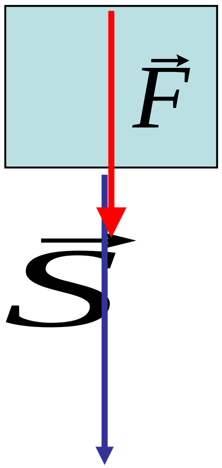
$$W_{F_2} = -FS = -m_1 gS$$

$$W_F = W_{F_1} + W_{F_2} = \left(m - m_1\right) gS > 0$$

After mass decrease at the top, the total work is positive

When drinking, we perform positive work!!!

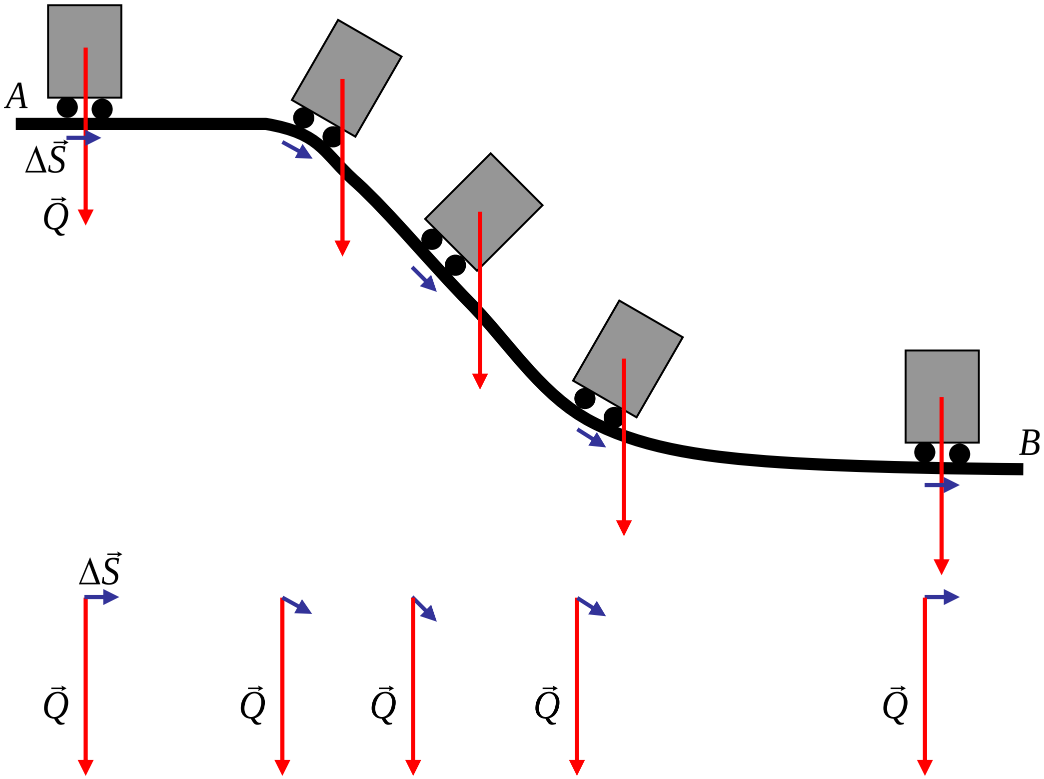
Pressing



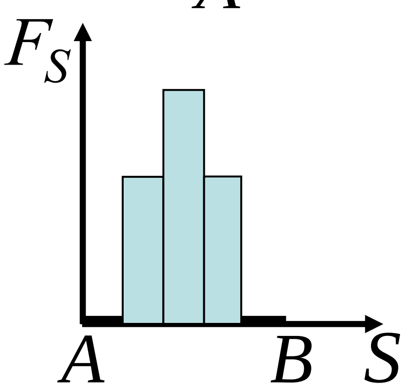
$$\alpha = 0^\circ \quad \Rightarrow \cos(\alpha) = 1 \Rightarrow W_F = FS$$

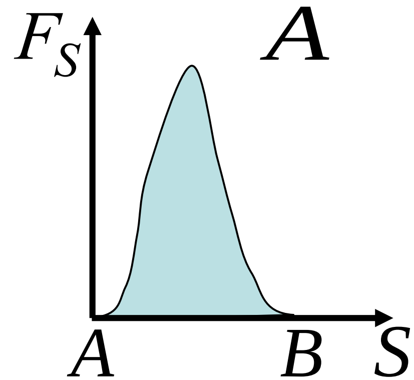
Pressing the remote control button we perform positive work!!!

$$W = \vec{F} \cdot \vec{S}$$



$$W = \sum_{A}^{B} \vec{F} \cdot \Delta\vec{S} \quad \Rightarrow \quad W = \int_{A}^{B} \vec{F} \cdot \mathrm{d}\vec{S}$$





WORK

The work is an integral of force and displacement scalar product.

$$W = \int_A^B \vec{F} \cdot \mathrm{d}\vec{S}$$

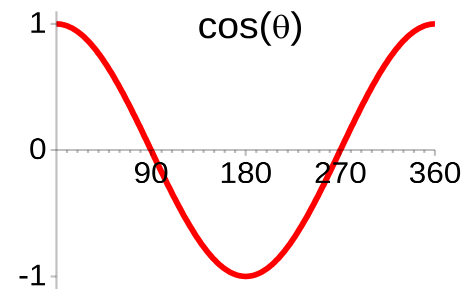
Work W is energy transferred to or from an object by means of a force acting on the object.

Energy transferred to the object is positive work.

Energy transferred from the object is negative work.

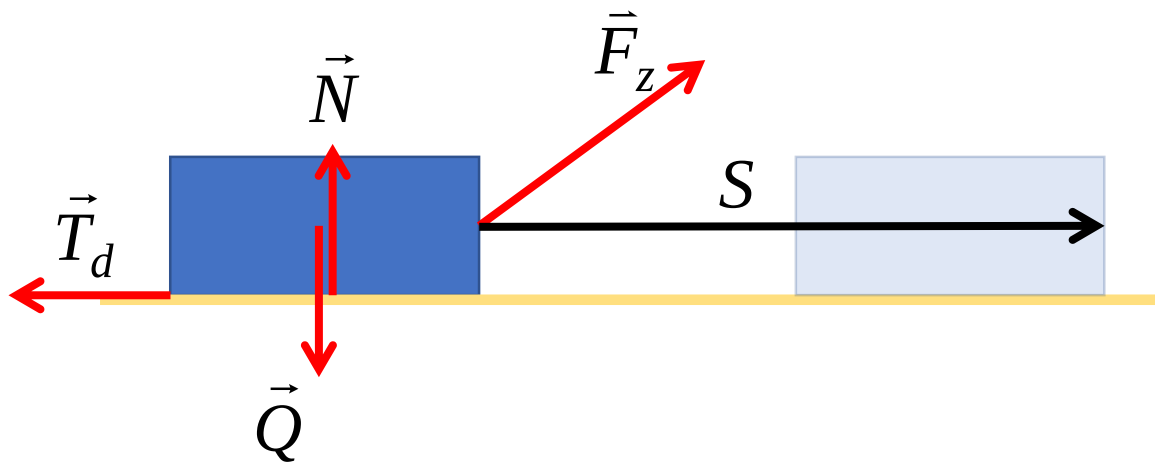
The Scalar Product (dot product)

$$R = \overrightarrow{A} \cdot \overrightarrow{B}$$



Example:

- work: $W = \vec{F} \cdot \vec{S}$,

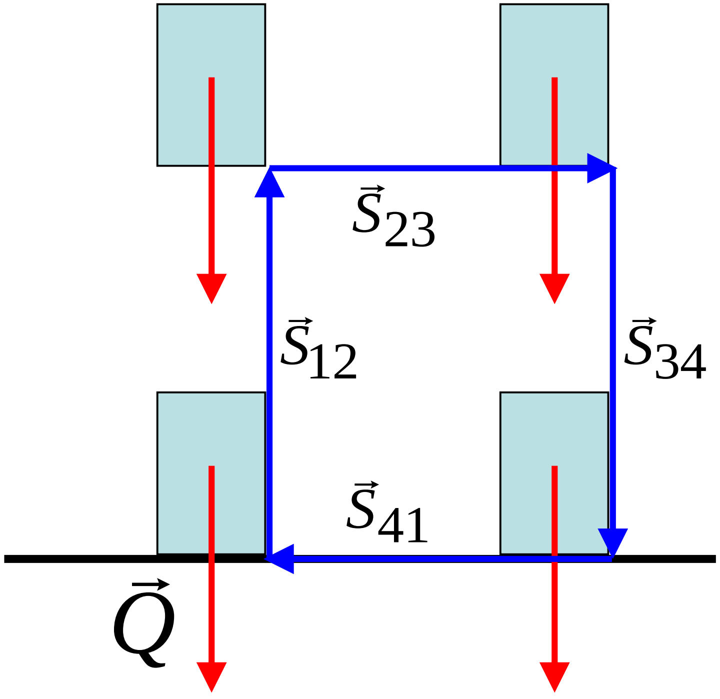


$$W_{F_z} > 0$$

$$W_Q = 0$$

$$W_N = 0$$

$$W_{T_d} < 0$$



$$W_{Q_{12}} = -mgS_{12}$$

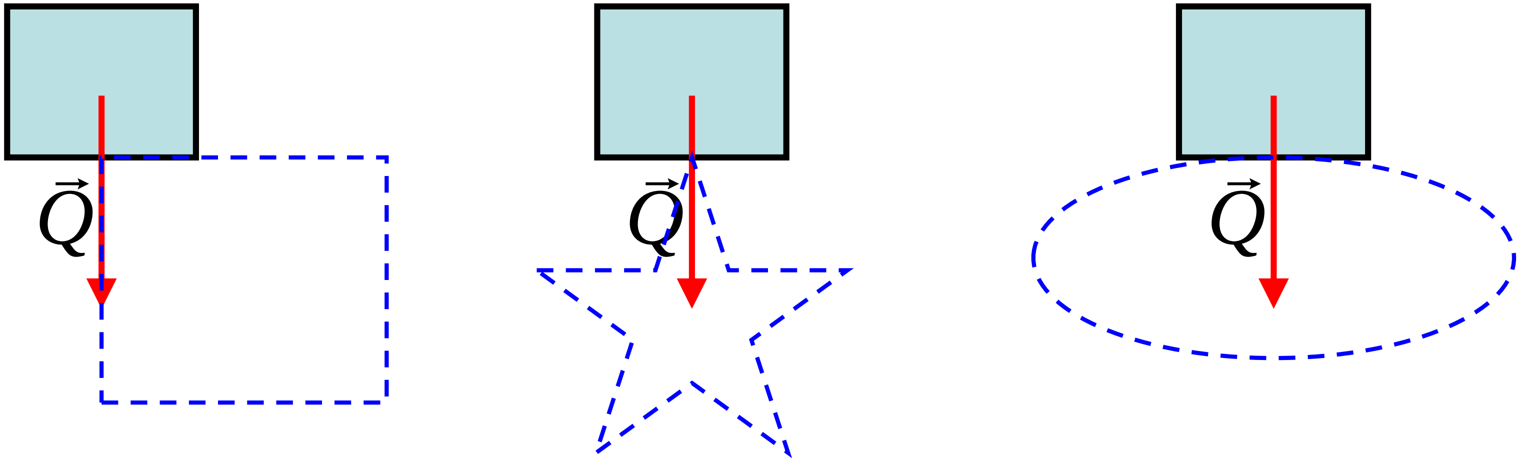
$$W_{Q_{23}} = 0$$

$$W_{Q_{34}} = mgS_{34}$$

$$W_{Q_{41}} = 0$$

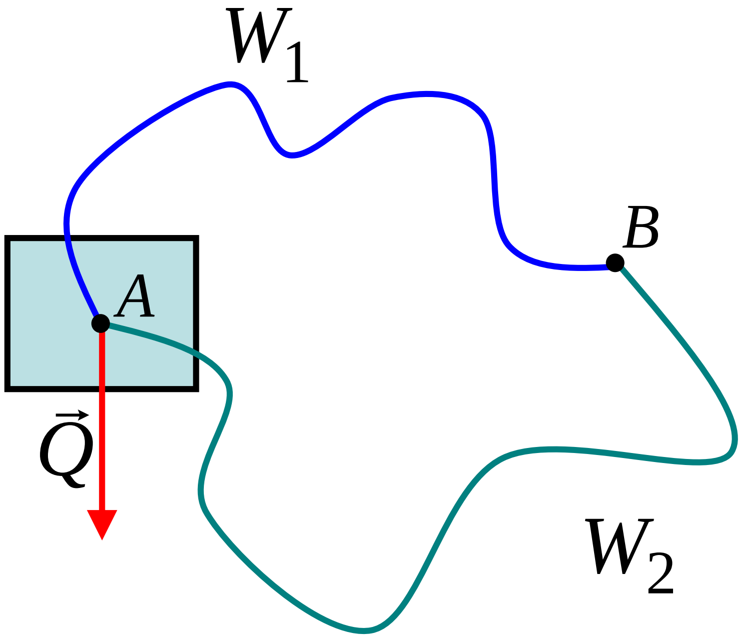
$$W_{Q_{12341}} = 0 \quad \left(S_{12} = S_{34}\right)$$

$$W = \vec{F} \cdot \vec{S} = FS\cos(\alpha) = QS_{12}\cos(180°) = -QS_{12}$$



$W = 0$ $\quad$ $W = 0$ $\quad$ $W = 0$

The net work done by a weight on a particle moving around any closed path is zero.



$$W_{ABA} = 0$$

$$W_{1_{AB}} + W_{2_{BA}} = 0$$

$$\left(W = \vec{F} \cdot \vec{S} \Rightarrow W_{2_{BA}} = -W_{2\,AB} \right)$$

$$W_{1_{AB}} - W_{2_{AB}} = 0$$

$$W_{1_{AB}} = W_{2_{AB}}$$

The work done by a weight on a particle mooving between any two points does not depend on the path taken by the particle.

A force,

which net work on any closed path is zero

(is independent on the path),

is said to be a **conservative force**.

Conservative force:

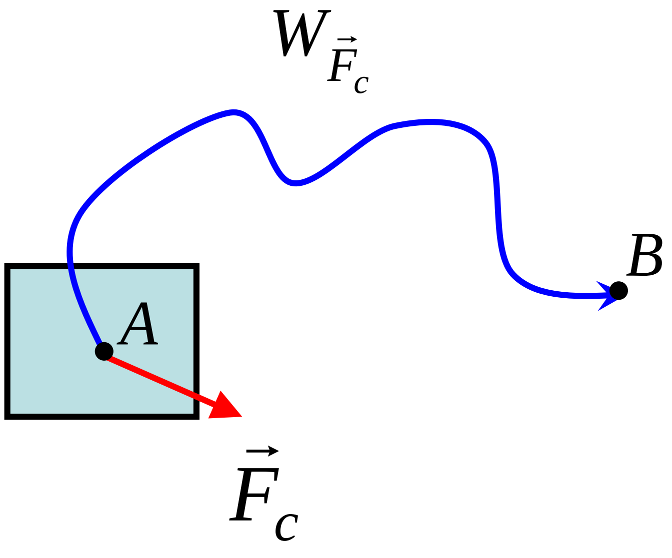
- gravitational force
- spring force
- Coulomb force

Nonconservative force :

- friction force $W_{T_k} < 0$
- drag force $W_{F_D} < 0$

$$\left.\begin{matrix} W_{T_k} < 0 \\ W_{F_D} < 0 \end{matrix}\right\} - \text{both forces are dissipative}$$

For conservative force it is possible to define the change of the **potential energy**



$$\Delta U = U_B - U_A = -W_{\vec{F}_c} = -\int_A^B \vec{F}_c d\vec{r}$$

$$U_A = 0 \Rightarrow U_B = -W_{\vec{F}_c} = -\int_A^B \vec{F}_c d\vec{r}$$

POTENTIAL ENERGY

The change ΔU in the potential energy

is defined to equal to the negative of the work

done by a conservative force

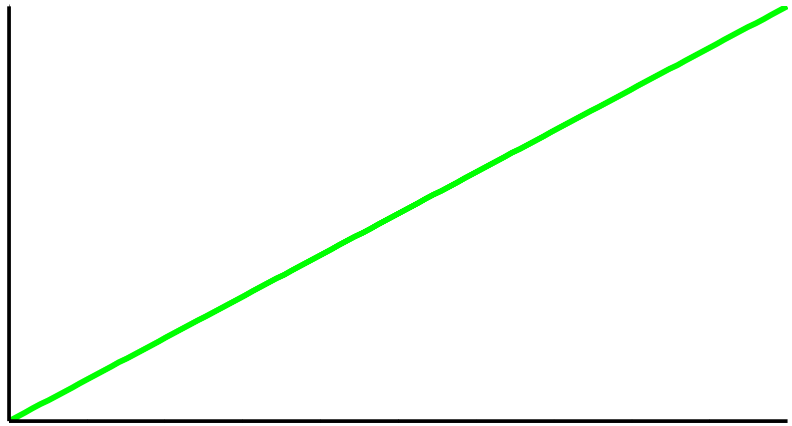
during the shift from an initial to a final state.

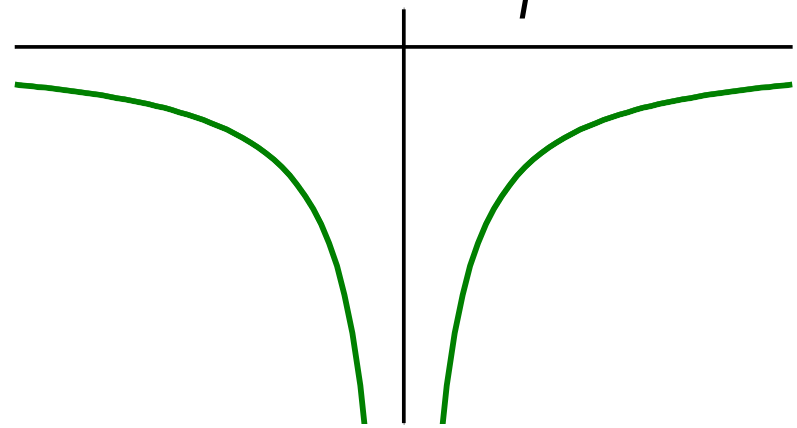
$$\Delta U = U_B - U_A = -W_{F_c} = -\int_A^B \vec{F}_c d\vec{r}$$

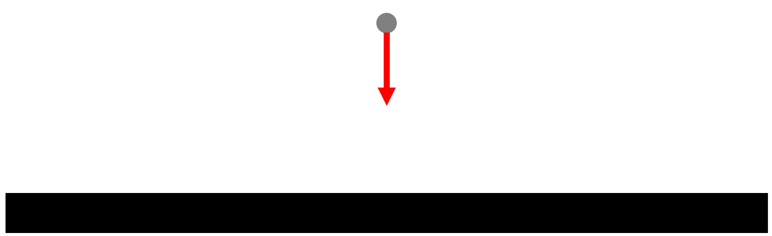
Conservative force		Potential energy
Gravitation	$m\vec{g}$	mgh
Grawitaion	$-G\dfrac{m_1m_2}{r^2}\dfrac{\vec{r}}{r}$	$-G\dfrac{m_1m_2}{r}$
Spring force	$-k\vec{x}$	$\frac{1}{2}kx^2$
Coulomb	$\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\dfrac{\vec{r}}{r}$	$\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r}$

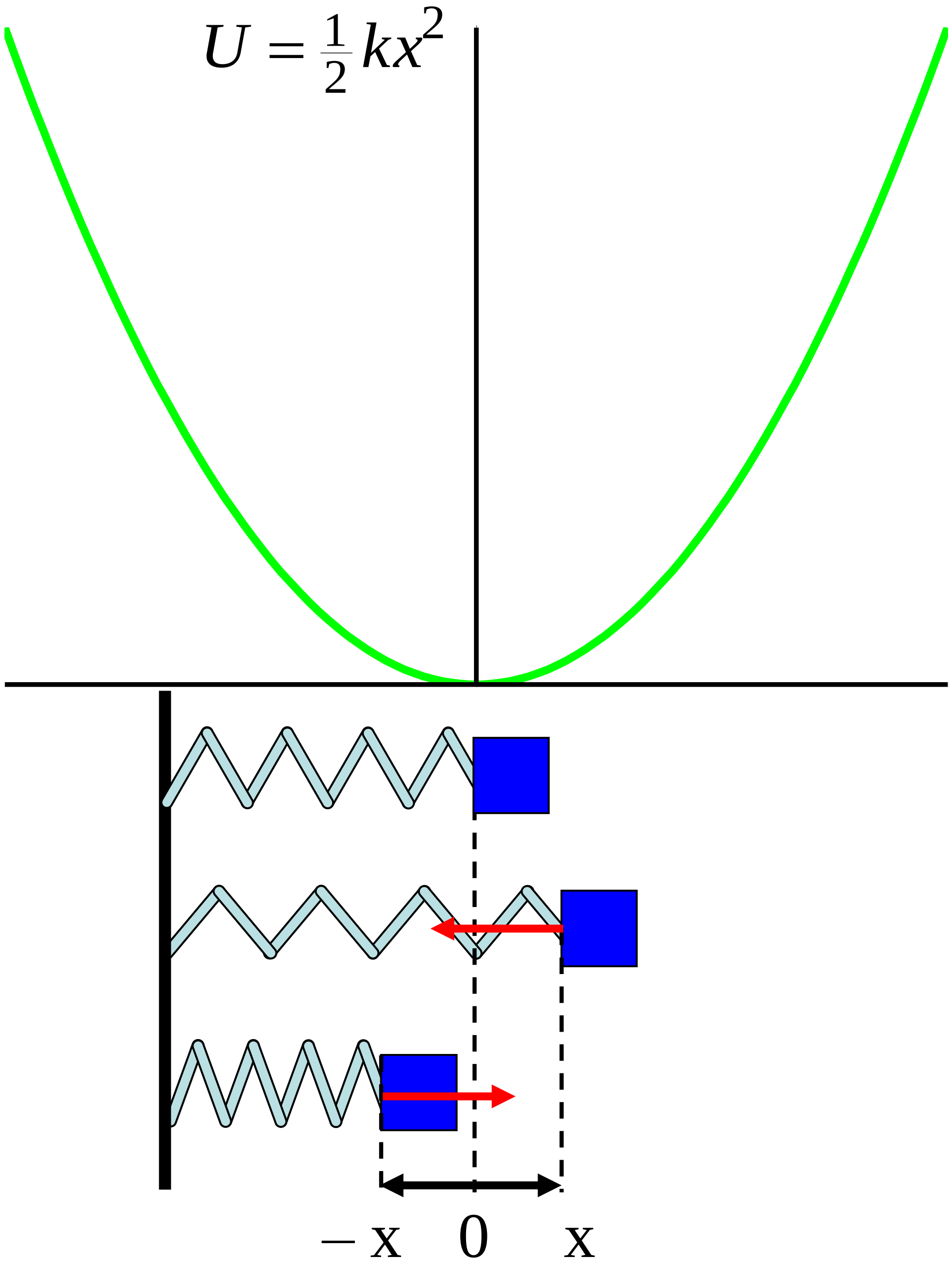
$$U = mgh$$

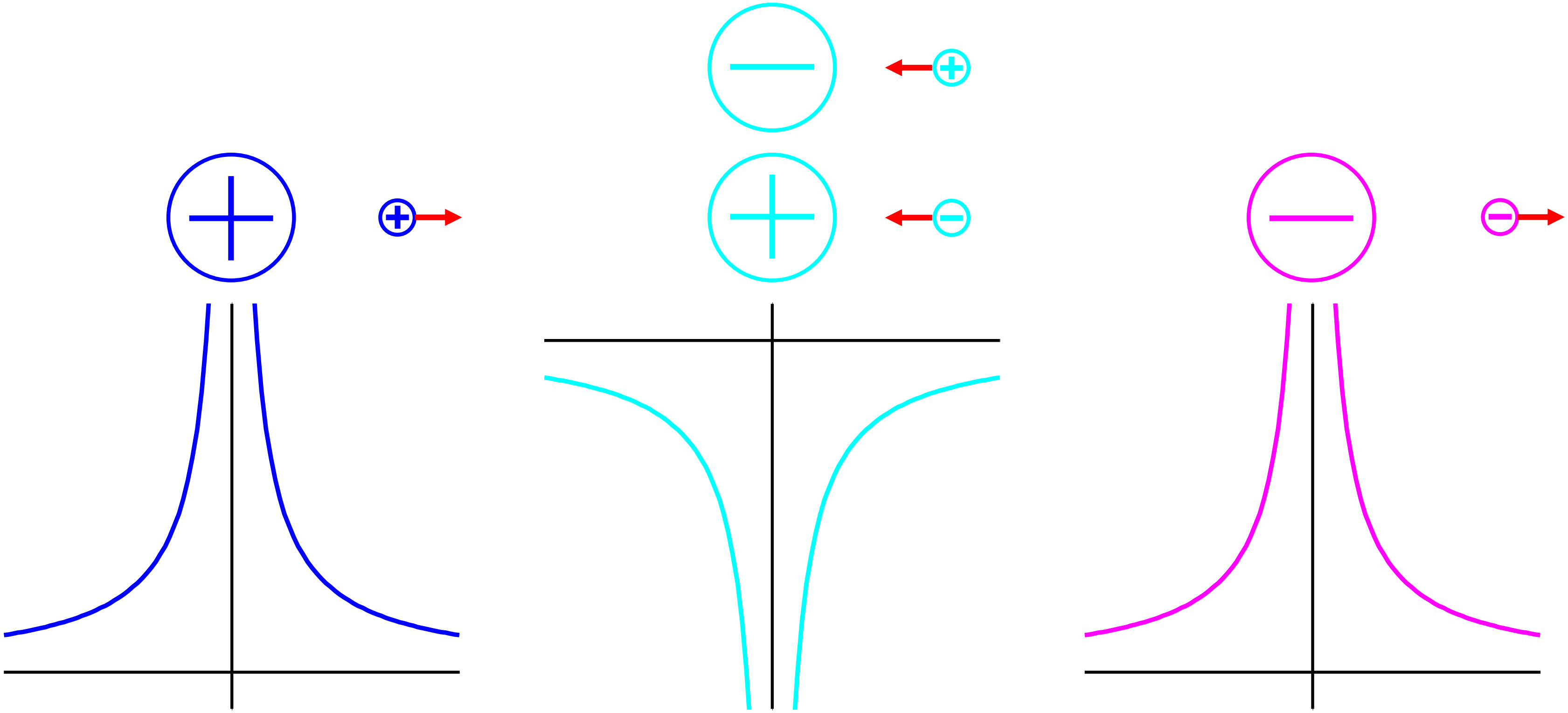
$$U = -G\frac{m_1 m_2}{r}$$








$U = \frac{1}{2}kx^2$
$-x$
0
x



$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

Body aim to achieve a state of minimum potential energy

Integral = Antiderivative

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x) \quad \Leftrightarrow y(x) = \int f(x)\mathrm{d}x$$

The integral is the inverse operation to the derivative – the antiderivative.

$$\Delta U = -\int_A^B \vec{F}_c d\vec{r}$$

$$\vec{F}_c = -\frac{\mathrm{d}U}{\mathrm{d}x}$$ (in one-dimensional motion)

$$\vec{F}_c = -\mathrm{grad}(U) = -\nabla U$$ (in 3D-dimensional motion)

$$U_{(x)} - \min \;\Rightarrow\; \frac{\mathrm{d}U}{\mathrm{d}x} = 0 \Rightarrow\; F_c = 0$$

$$U_{(x)} - \max \;\Rightarrow\; \frac{\mathrm{d}U}{\mathrm{d}x} = 0 \Rightarrow\; F_c = 0$$

$$U_{(x)} - \mathrm{const} \;\Rightarrow\; \frac{\mathrm{d}U}{\mathrm{d}x} = 0 \Rightarrow\; F_c = 0$$

EQUILIBRIUM

$$U_{(x)} - \mathrm{min} \cup \mathrm{max} \cup \mathrm{const}$$

$$\Leftrightarrow \quad \frac{\mathrm{d}U}{\mathrm{d}x} = 0 \quad \Leftrightarrow \quad F_c = 0$$

$\Leftrightarrow$ the body remains at rest

$\Leftrightarrow$ the body remains at equilibrium

- stable $U_{(x)} - \mathrm{min}$

- unstable $U_{(x)} - \mathrm{max}$

- neutral $U_{(x)} - \mathrm{const}$

The body is at **equilibrium** when:

- net force is zero

$$\vec{F}_w = \vec{0} \text{ and } \vec{M}_w = \vec{0}$$

(dynamic equilibrium condition)

- remains at the extremum of the potential energy

$$\frac{\mathrm{d}U}{\mathrm{d}x} = 0$$

(energetic equilibrium condition)

KINETIC ENERGY

$$W = \int_A^B \vec{F}_w \mathrm{d}\vec{r}$$

$$\vec{F}_w = \frac{\mathrm{d}\,\vec{\mathrm{p}}}{\mathrm{d}t} = \frac{\mathrm{d}\left(m\vec{\mathrm{V}}\right)}{\mathrm{d}t} = m\frac{\mathrm{d}\vec{V}}{\mathrm{d}t}$$

$$\vec{V} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \Rightarrow \mathrm{d}\vec{r} = \vec{V}\mathrm{d}t$$

$$W = \int_A^B m\frac{\mathrm{d}\vec{V}}{\mathrm{d}t}\vec{V}\mathrm{d}t$$

$$W = m\int_A^B \vec{V}\mathrm{d}\vec{V}$$

$$W = m\int_{A}^{B} \vec{V}\mathrm{d}\vec{V}$$

$$W = \tfrac{1}{2} m V_{\mathrm{B}}^2 - \tfrac{1}{2} m V_A^2$$

$$K = \tfrac{1}{2} m V^2$$

$$W = K_B - K_A$$

$$\Delta K = W_{\vec{F}_w} = \int_{A}^{B} \vec{F}_w \mathrm{d}\vec{r}$$

KINETIC ENERGY

The change ΔK in the kinetic energy

is defined to equal to the work

done by a conservative force

during the shift from an initial to a final state.

$$\Delta K = K_B - K_A = W_{\vec{F}_w} = \int_A^B \vec{F}_w \mathrm{d}\vec{r}$$

When acting forces are conservative:

$$\vec{F}_w = \vec{F}_c$$

$$\int_A^B \vec{F}_w \mathrm{d}\vec{r} = \int_A^B \vec{F}_c \mathrm{d}\vec{r}$$

$$W_{\vec{F}_w} = W_{\vec{F}_c}$$

$$\Delta K = -\Delta U$$

$$K_B - K_A = -(U_B - U_A)$$

$$K_B + U_B = K_A + U_A$$

$$E = K + U$$

$$E_B = E_A$$

$$E = \text{const}$$

PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

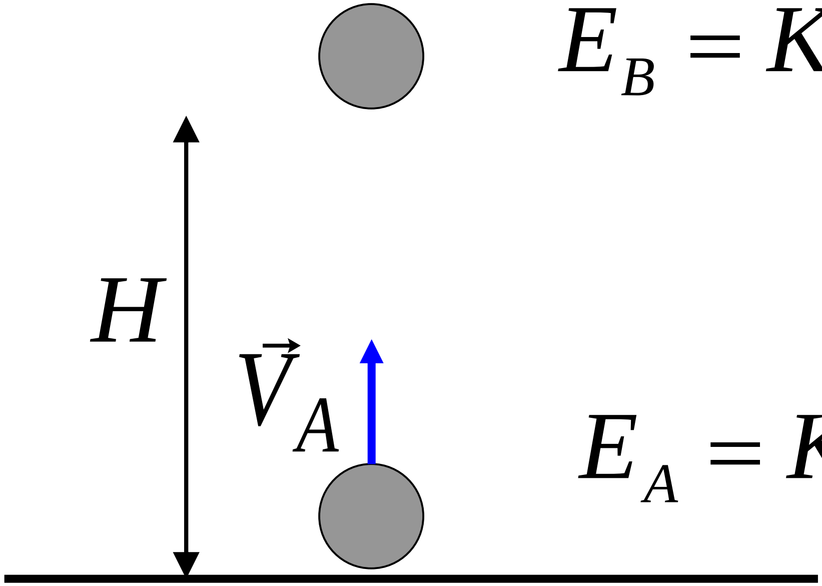
If only a conservative forsec within the system does work,

then the total mechanical energy E of the system,

the total sum of its kinetic K an its potential U energies,

cannot change

$$\vec{F}_w = \vec{F}_c \quad \Rightarrow E = K + U = \text{const}$$

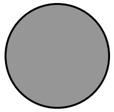


$$E_B = K_B + U_B = mgH \qquad (K_B = 0)$$

$$E_A = K_A + U_A = \frac{mV_A^2}{2} \qquad (U_A = 0)$$

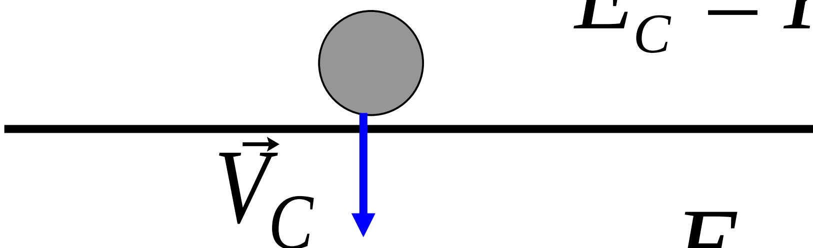
$$E_B = E_A \quad \Rightarrow mgH = \frac{mV_A^2}{2}$$

$$H = \frac{V_A^2}{2g}$$



$$E_B = K_B + U_B = mgH \qquad (K_B = 0)$$

$$E_C = K_C + U_C = \frac{mV_C^2}{2} \qquad (U_C = 0)$$

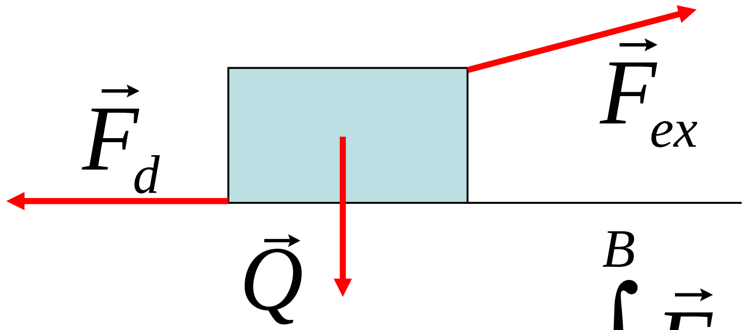


$$E_C = E_B \quad \Rightarrow \frac{mV_C^2}{2} = mgH$$

$$V_C = \sqrt{2gH}$$

$$V_C = V_A \quad \Leftarrow \quad V_C = \sqrt{2g\frac{V_A^2}{2g}}$$

When on a body, except conservative foces $\vec{F}_c$, acts dissipative forces $\vec{F}_d$ and external forces $\vec{F}_{ex}$:



$$\vec{F}_w = \vec{F}_c + \vec{F}_d + \vec{F}_{ex}$$

$$\int_A^B \vec{F}_w \mathrm{d}\vec{r} = \int_A^B \vec{F}_c \mathrm{d}\vec{r} + \int_A^B \vec{F}_d \mathrm{d}\vec{r} + \int_A^B \vec{F}_{ex} \mathrm{d}\vec{r}$$

$$W_{\vec{F}_w} = W_{\vec{F}_c} + W_{\vec{F}_d} + W_{\vec{F}_{ex}}$$

$$\Delta K = -\Delta U - \Delta E_{\mathrm{int}} + W_{\vec{F}_{ex}}$$

$$\Delta K + \Delta U + \Delta E_{\mathrm{int}} = W_{\vec{F}_{ex}}$$

PRINCIPLE OF TOTAL ENERGY CONSERVATION

$$\Delta E_{tot} = \Delta K + \Delta U + \Delta E_{\text{int}} = W_{\vec{F}_{ex}}$$

The change in the total energy of the system is equal to the work done by an external force.

PRINCIPLE OF TOTAL ENERGY CONSERVATION

$$W_{F_{ex}} = 0 \Rightarrow \Delta E_{tot} = \Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

In an isolated system,

(system without external foces)

energy may be transferred from one type to another,

but the total energy E_{tot} of the system

always remains constant.

INTERNAL ENERGY

$$\Delta E_{\text{int}} = -W_{\vec{F}_d}$$

1. The change of the internal energy is equal to the negative of the dissipative force work.

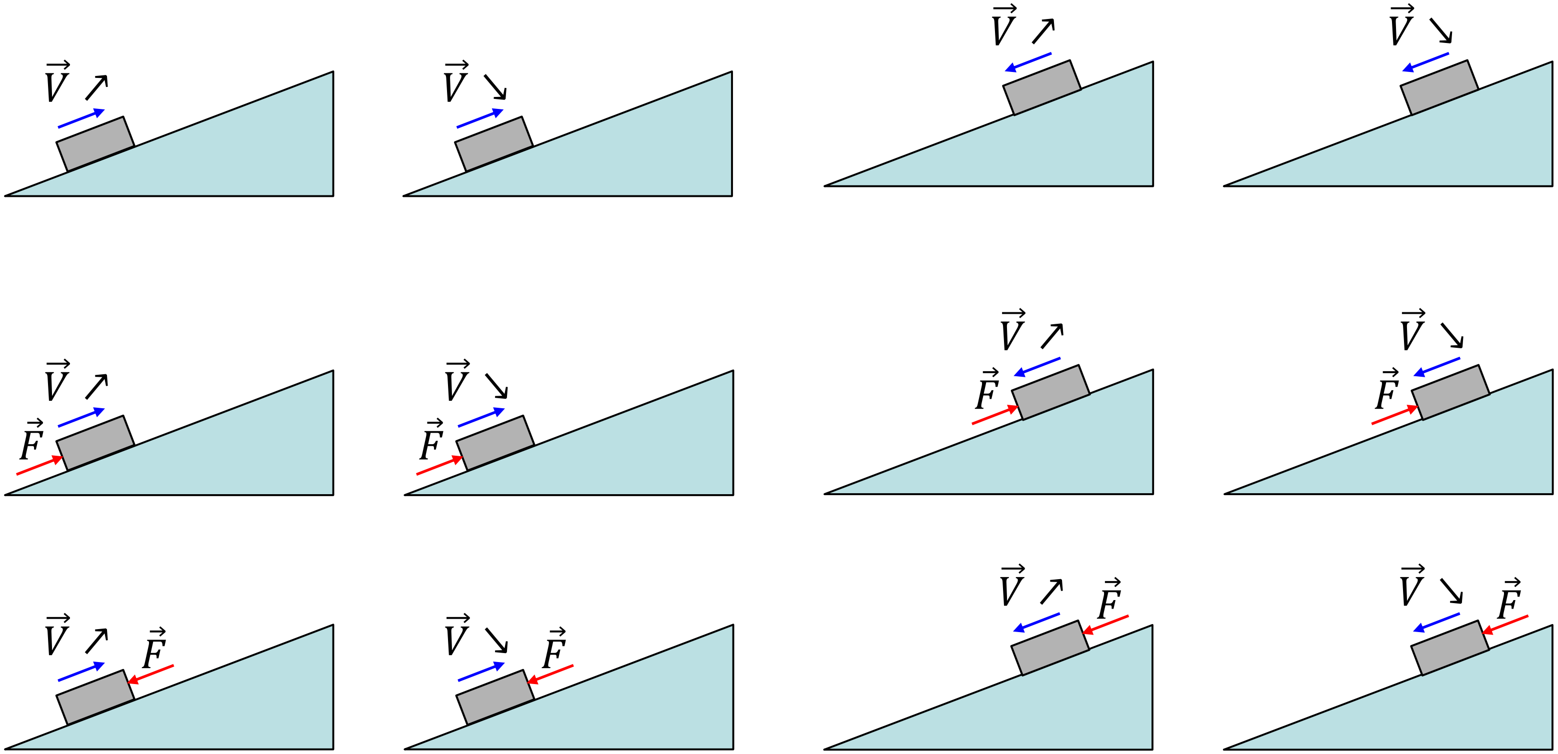
2. Work done by torque or drag foce $\left(W_{\vec{F}_d}\right)$ always increase the internal energy of the system $(\Delta E_{int} > 0)$.

3. Internal energy could be observed as:
   - body and environment heating,
   - deformation,
   - sound,
   - light...

Discuss energy transfers occurring in each of the following 12 situations:

a) not taking into account the occurrence of frictional force

b) taking into account the occurrence of friction force

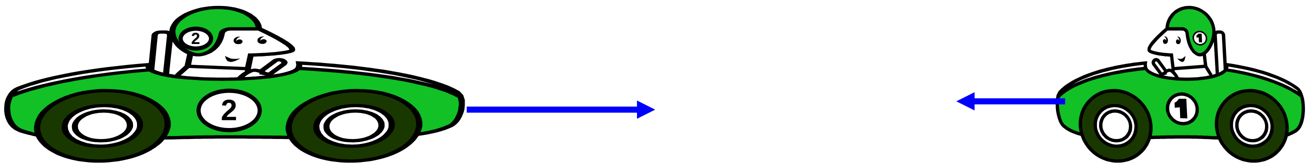


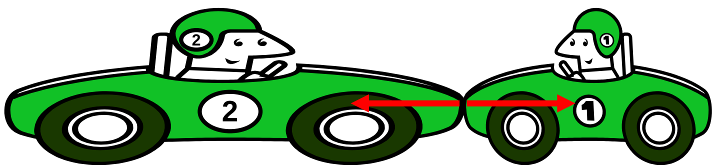
1) Which of the presented cases can not occur in reality??

2) In which of the presented cases the speed could be constant?

Discuss energy transfers in such a case.

COLLISIONS





$$\vec{F}_{21} = -\vec{F}_{12}$$

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\vec{p}_1}{\mathrm{d}t} = -\frac{\mathrm{d}\vec{p}_2}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\vec{p}_1}{\mathrm{d}t} + \frac{\mathrm{d}\vec{p}_2}{\mathrm{d}t} = \vec{0}$$

$$\frac{\mathrm{d}\left(\vec{p}_1 + \vec{p}_2\right)}{\mathrm{d}t} = \vec{0} \qquad \Rightarrow \qquad \vec{p}_1 + \vec{p}_2 = \overrightarrow{const}$$

PRINCIPLE OF

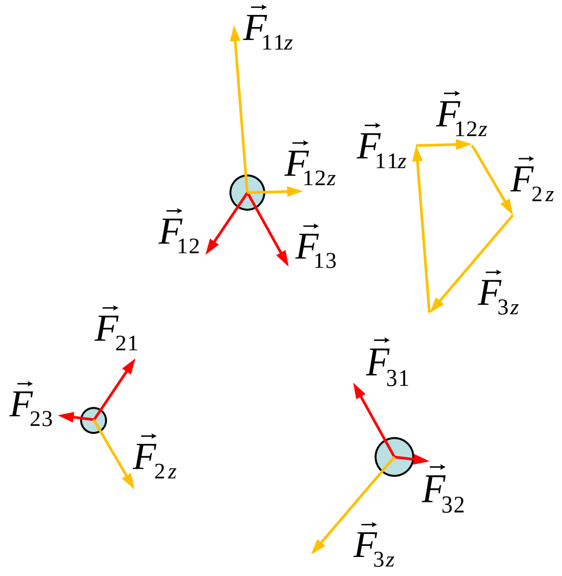
MOMENTUM CONSERVATION

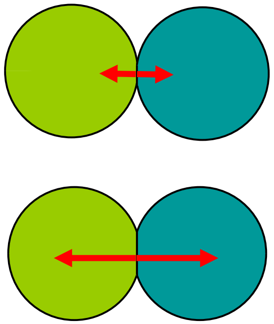
If there is no external foce or net external force is zero ,

the net linear momentum of the system

cannot change.

$$\vec{F}_{wz} = \vec{0} \Rightarrow \vec{p}_c = \sum \vec{p}_i = \overrightarrow{\text{const}}$$





If in collisions acting forces are conservative then the mechanical energy is constant.

$$F_{21} = F_{12} = -kx$$

The collisions is then said to be **elastic** .

$$\vec{p}_{1A} + \vec{p}_{2A} = \vec{p}_{1B} + \vec{p}_{2B} \qquad (MC)$$

$$\frac{m\vec{V}_{1A}}{2} + \frac{m\vec{V}_{2A}}{2} = \frac{m\vec{V}_{1B}}{2} + \frac{m\vec{V}_{2B}}{2} \qquad (MEC)$$

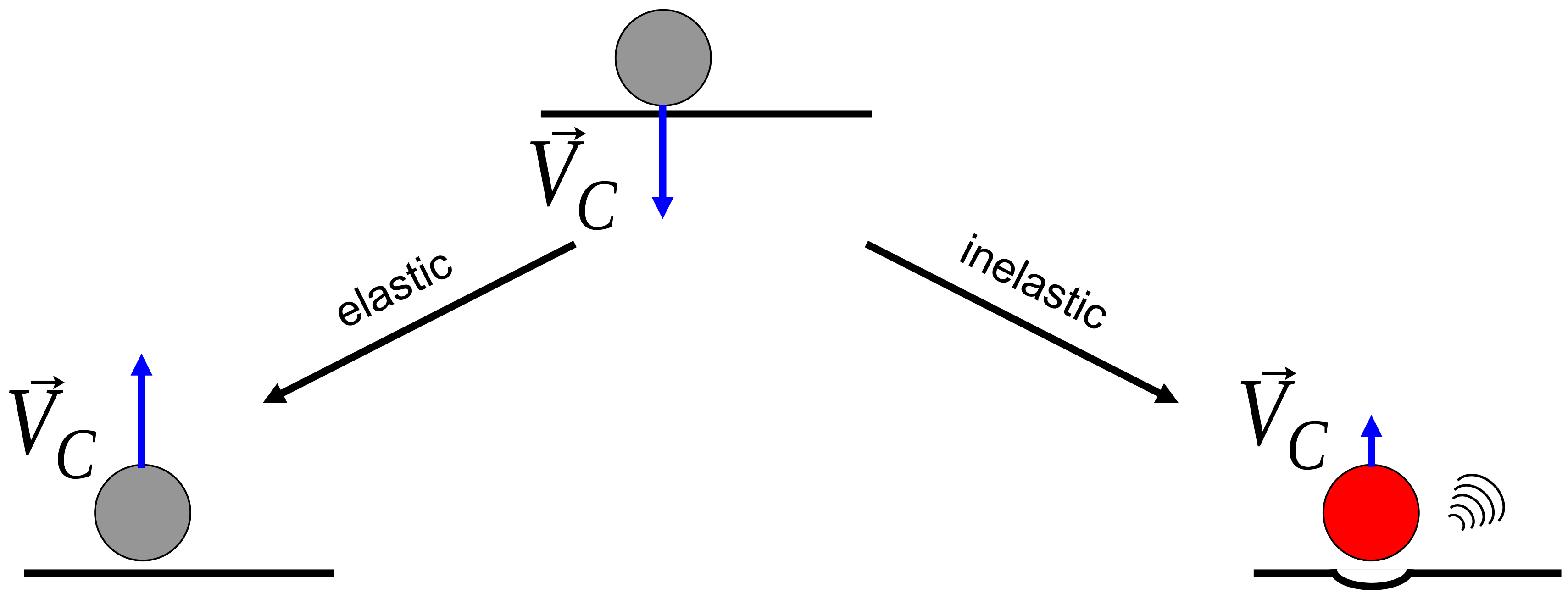
$$(\Delta U = 0)$$

If in collisions acting forces are nonconservative then the total energy is constant.

The collisions is then said to be **inelastic**.

$$\vec{p}_{1A} + \vec{p}_{2A} = \vec{p}_{1B} + \vec{p}_{2B} \qquad (MC)$$

$$\frac{m\vec{V}_{1A}}{2} + \frac{m\vec{V}_{2A}}{2} = \frac{m\vec{V}_{1B}}{2} + \frac{m\vec{V}_{2B}}{2} + \Delta E_{\text{int}} \qquad (TEC)$$



In elastic collisions:

- total mechanical energy is constant

In inelastic collisions:

- total mechanical energy decreases and follows:
- heating,
- deformation,
- sound generation,
- ligt generation.