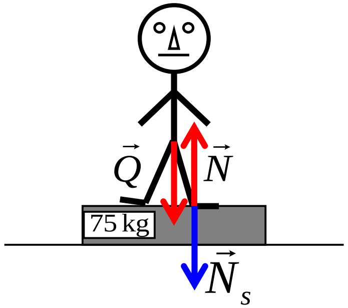
INERTIAL FORCES

(pseudo forces)

- a fictitious forces that „acts” on all masses whose motion is described using a non-inertial frame of reference

BODY ON A SCALE



$$m = 75\text{kg}$$

$$\vec{Q} \Rightarrow \vec{N}_s \Rightarrow \vec{N} = -\vec{N}_s$$

$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} = \vec{0}$$

$$-Q + N = 0$$

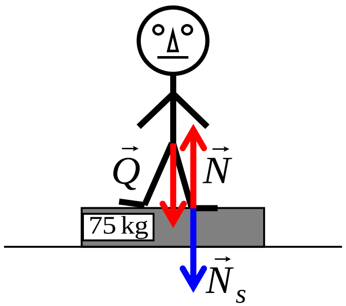
$$N = Q$$

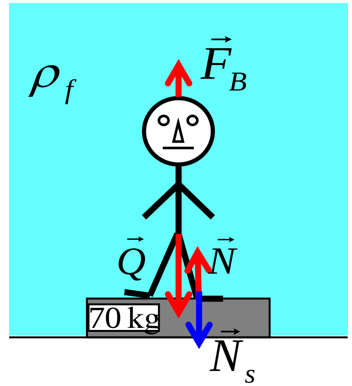
$$N = mg$$

$$N_s = mg$$

$$\frac{N_s}{g} = m$$

A weighting scale indicates the force applied, calibrated in kilograms!!!!!





$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} + \vec{F}_B = \vec{0}$$

$$-Q + N + F_B = 0$$

$$N = Q - F_B$$

$$N_s = N = mg - \rho_f V_d g$$

$$N_s = Q = mg$$

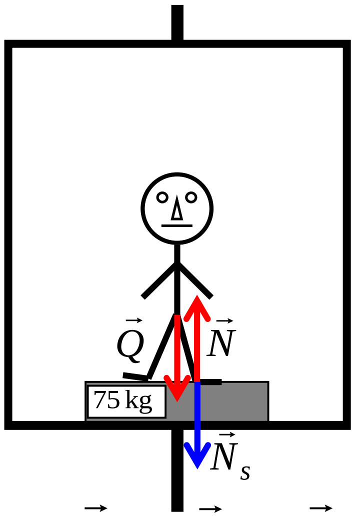
$$N_s = mg - m_{fd} g$$

$$N_s = mg - Q_{fd}$$

A body fully or partially immersed in a fluid is buoyed up by e force equal to the weight of the fluid that the body displaced. (Archimedes' principle)

BODY ON A SCALE IN A LIFT

$m = 75\text{kg}$



$$\vec{V} = \overrightarrow{const}$$

$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} = \vec{0}$$

$$-Q + N = 0$$

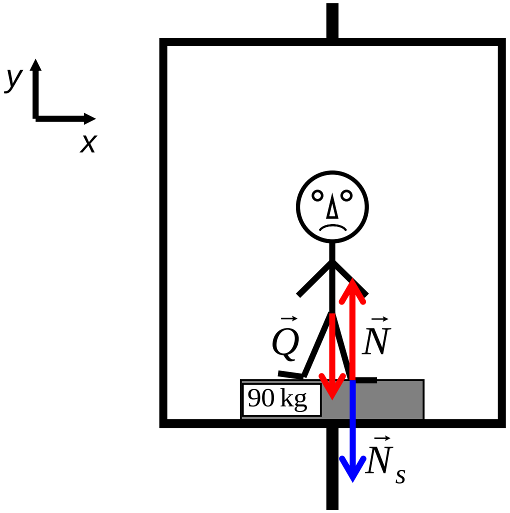
$$N = Q$$

$$N = mg$$

$$N_s = N = mg$$

$$\frac{N_s}{g} = m$$

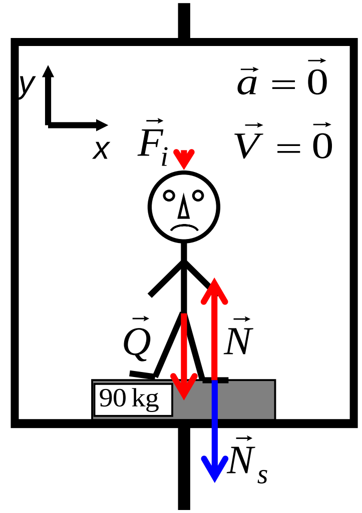
LINEAR ACCELERATION



$$m = 75\text{kg}$$

$$\vec{a}_u = \overrightarrow{const}$$

$$a_u = 2\,\text{m/s}^2$$



$$\vec{a} \neq \vec{0} \Rightarrow \vec{F}_w = m\vec{a}$$

$$\vec{Q} + \vec{N} = m\vec{a}$$

$$-Q + N = ma_u$$

$$N = Q + ma_u$$

$$N = mg + ma_u$$

$$N_s = m(g + a_u)$$

$$\frac{N_s}{g} = m\left(1 + \frac{a_u}{g}\right)$$

$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} + \vec{F}_i = \vec{0}$$

$$-Q + N - F_i = 0$$

$$F_i = -Q + N$$

$$N = N_s = mg + ma_u$$

$$F_i = -mg + mg + ma_u$$

$$F_i = ma_u$$

$$\vec{F}_i = -m\vec{a}_u$$

Real forces - arise from any physical interaction between two objects

Fictitious forces - result from the acceleration of the reference frame

In inertial frame of reference:

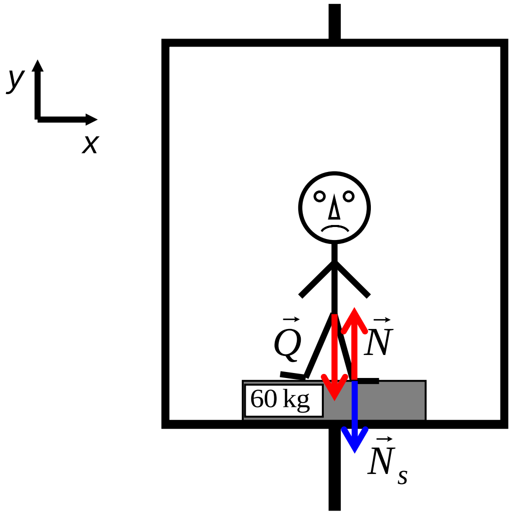
Newton's laws (I and II) are satisfied taking into accounts only real forces

In non-inertial frame of reference:

Newton's laws (I and II) are satisfied taking into accounts both real and fictitious forces

In non-inertial reference frame, moving with a linear acceleration $\vec{a}_u$, appears a fictitious force – **an inertial pseudo force**

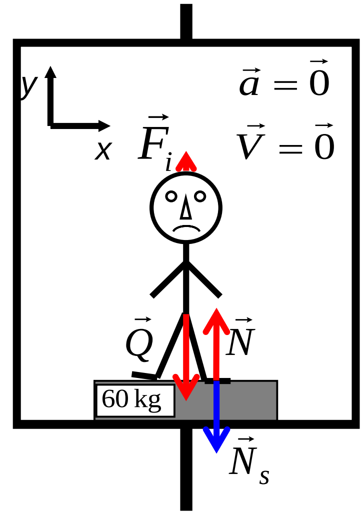
$$\vec{F}_i = -m\vec{a}_u$$



$$m = 75\text{kg}$$

$$\vec{a}_u = \overrightarrow{const}$$

$$a_u = 2\,\text{m/s}^2$$



$$\vec{a} \neq \vec{0} \Rightarrow \vec{F}_w = m\vec{a}$$

$$\vec{Q} + \vec{N} = m\vec{a}$$

$$-Q + N = m(-a_u)$$

$$N = Q - ma_u$$

$$N = mg - ma_u$$

$$N_s = m(g - a_u)$$

$$\frac{N_s}{g} = m(1 - \frac{a_u}{g})$$

$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} + \vec{F}_i = \vec{0}$$

$$-Q + N + F_i = 0$$

$$F_i = Q - N$$

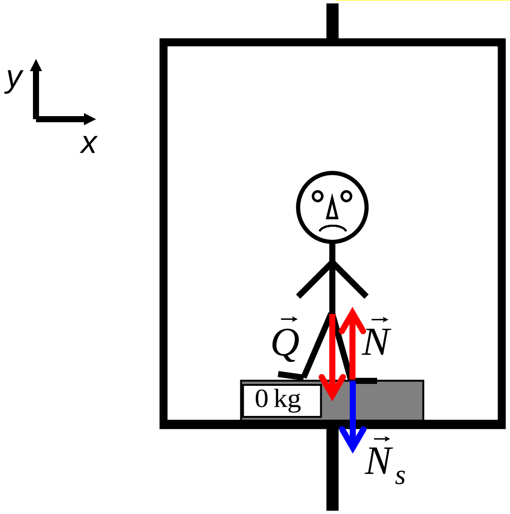
$$N = N_s = mg - ma_u$$

$$F_i = mg - (mg - ma_u)$$

$$F_i = ma_u$$

$$\vec{F}_i = -m\vec{a}_u$$

WEIGHTLESSNESS



$$m = 75\text{kg}$$

$$\vec{a}_u = \overrightarrow{const}$$

$$a_u = g = 9.81\text{m/s}^2$$

Is an absence of stress and strain resulting from externally applied mechanical contact-forces, typically normal forces

$$\vec{a} \neq \vec{0} \Rightarrow \vec{F}_w = m\vec{a}$$

$$\vec{Q} + \vec{N} = m\vec{a}$$

$$-Q + N = m(-a_u)$$

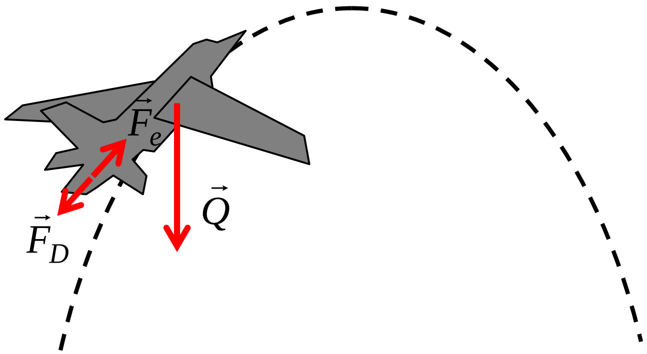
$$N = Q - ma_u$$

$$N = mg - ma_u$$

$$N_s = m(g - a_u)$$

$$N_s = 0$$

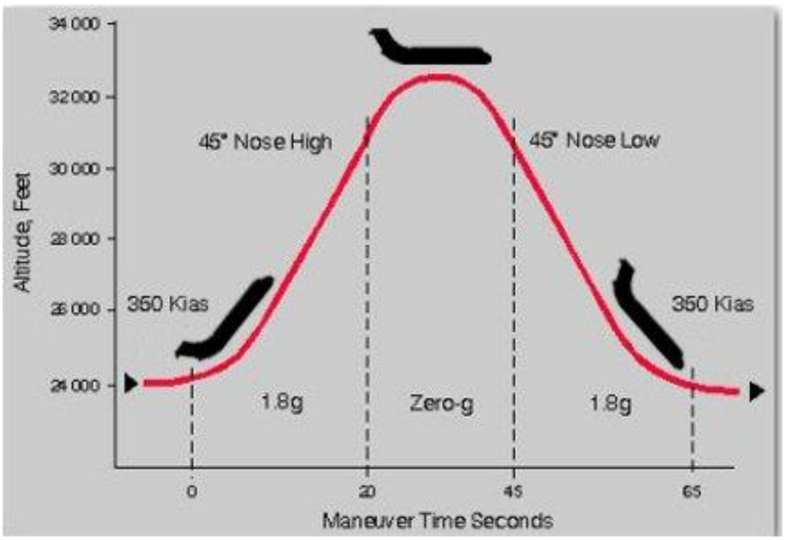
$$\frac{N_s}{g} = 0$$



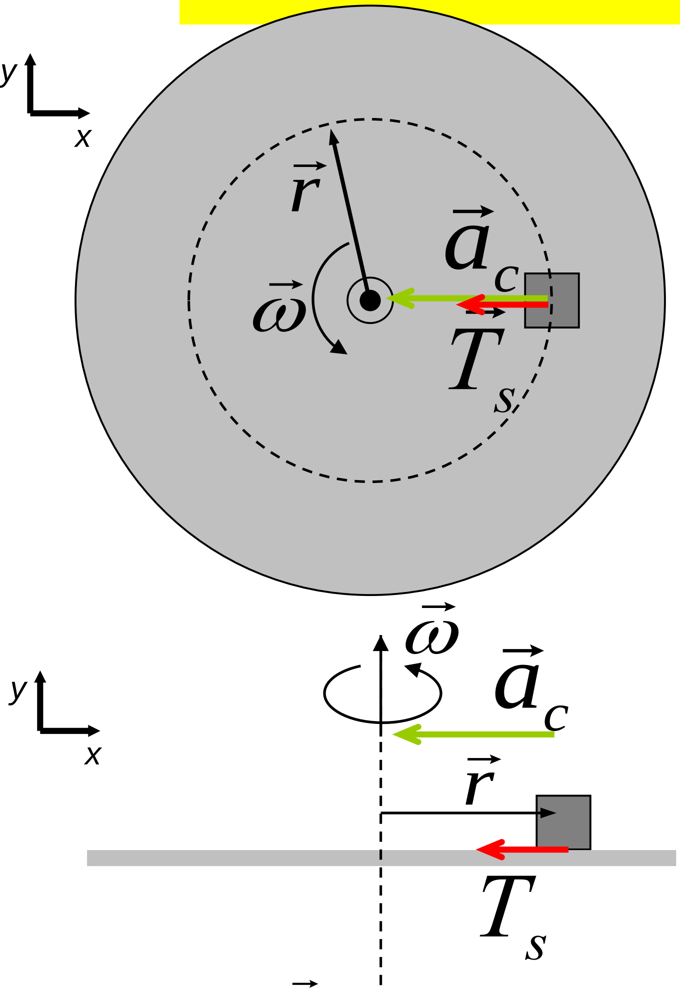
$$\vec{F}_e = \vec{F}_D \Rightarrow \vec{Q} = m\vec{a}$$

$$m\vec{a} = m\vec{g}$$

$$\vec{a} = \vec{g}$$



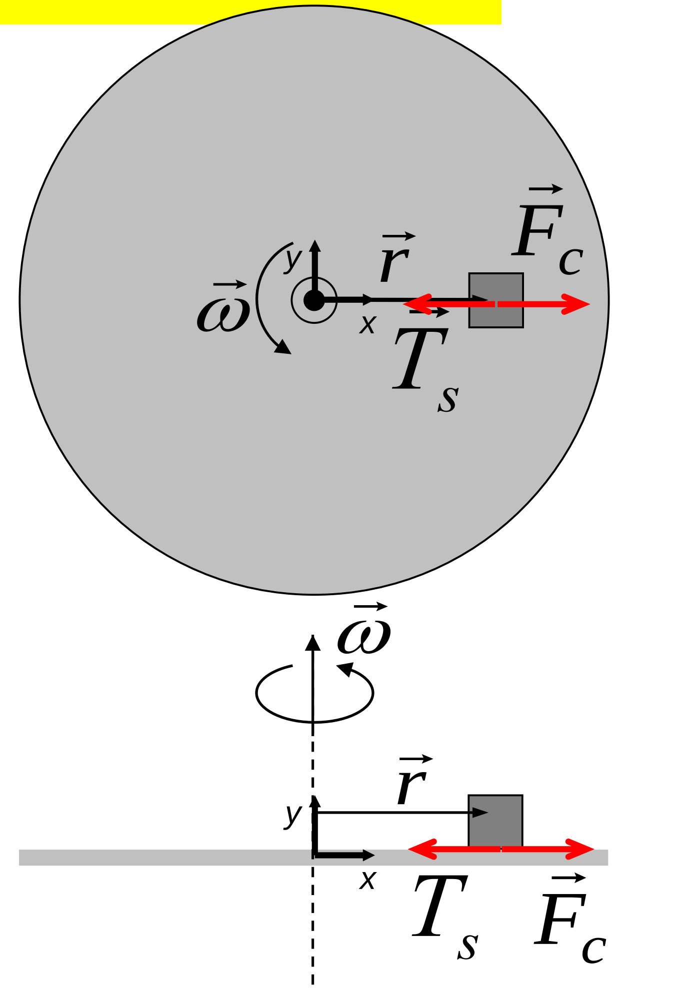
ROTATING REFERENCE FRAME



$$\vec{F}_w = m\vec{a}$$

$$-T_s = -ma_c$$

$$T_s = m\frac{V^2}{r}$$



$$\vec{F}_w = \vec{0}$$

$$-T_s + F_c = 0$$

$$F_c = m\frac{V^2}{r} = ma_c$$

$$\vec{F}_c = -m\vec{a}_c$$

$$F_c = m\frac{V^2}{r}$$

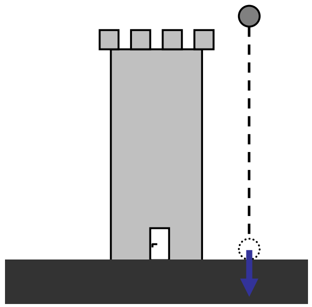
$$(V = \omega r)$$

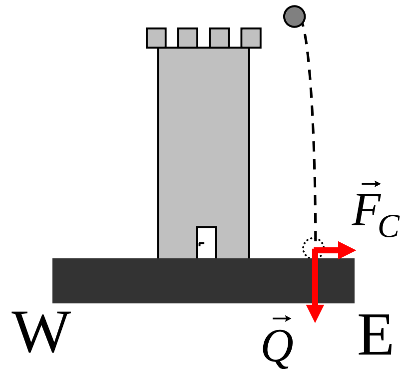
$$F_c = m\omega^2 r$$

$$\vec{F}_c = -m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

In non-inertial reference frame, rotating with a centripetal acceleration $\vec{a}_c$, appears a fictitious force – **a centrifugal pseudo force**

$$\vec{F}_c = -m\vec{a}_c$$





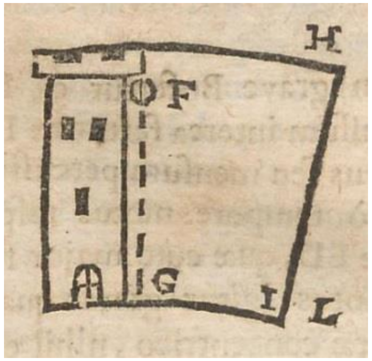
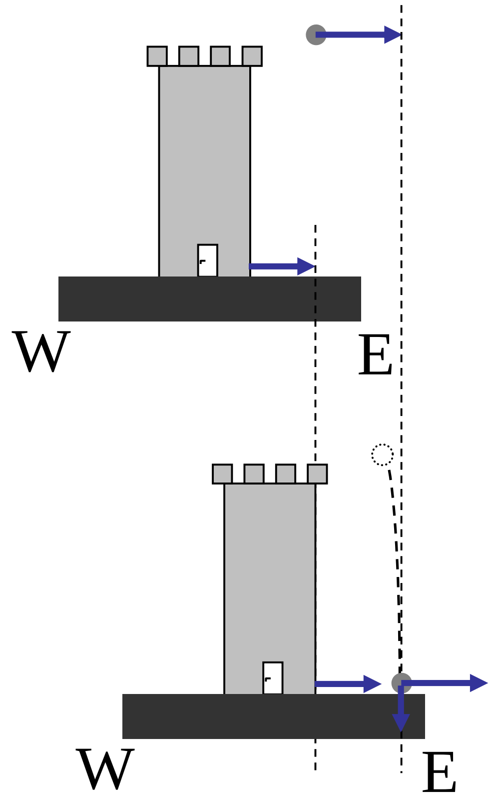
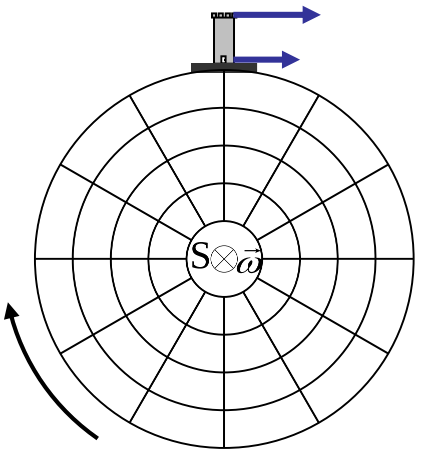


Image from *Cursus seu Mundus Mathematicus* (1674) of C.F.M. Dechales, showing how a ball should fall from a tower on a rotating Earth. The ball is released from *F*. The top of the tower moves faster than its base, so while the ball falls, the base of the tower moves to *I*, but the ball, which has the eastward speed of the tower's top, outruns the tower's base and lands further to the east at *L*.
source: Wikipedia



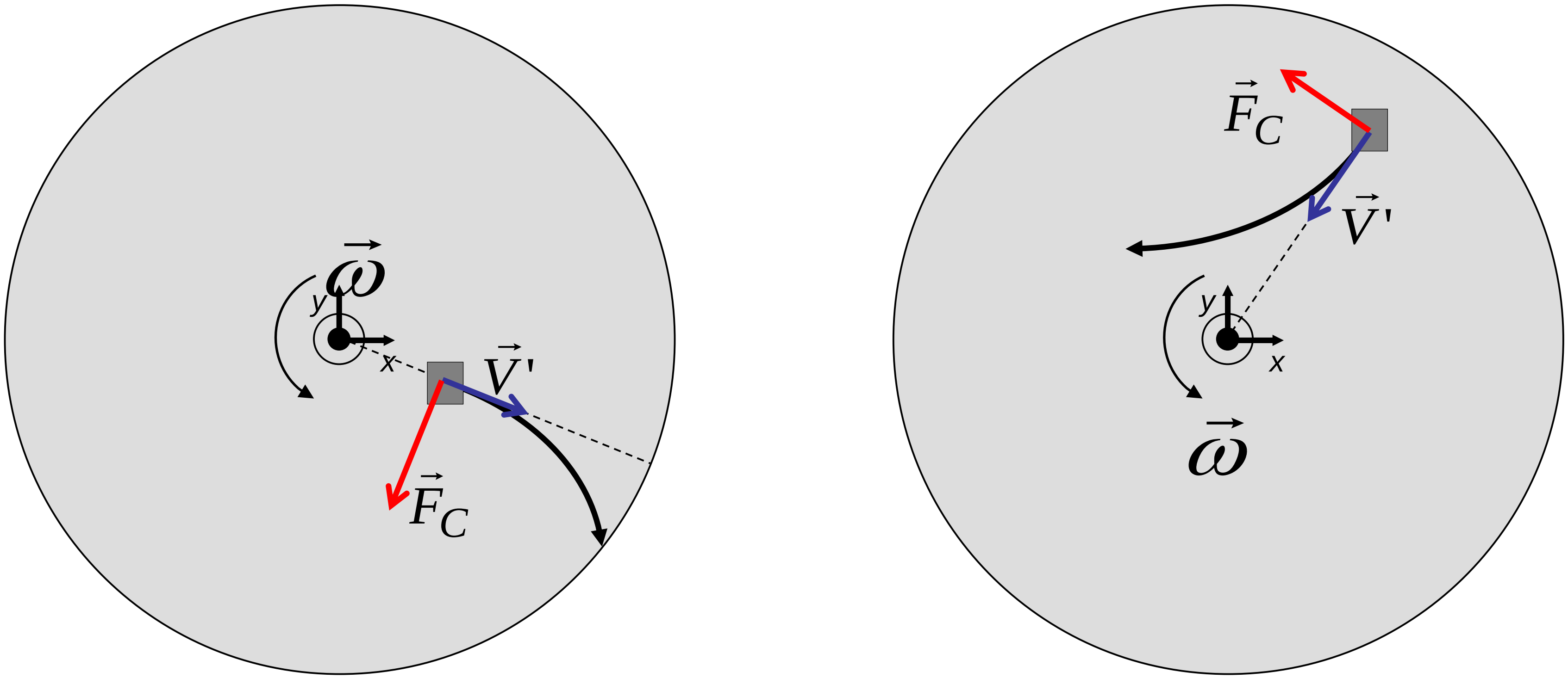


Rotating Earth from the South Pole

In non-inertial reference frame, rotating with a constant angular velocity $\vec{\omega}$, for a body moving with linear velocity $\vec{V}$, appears a fictitious force – **a Coriolis pseudo force**

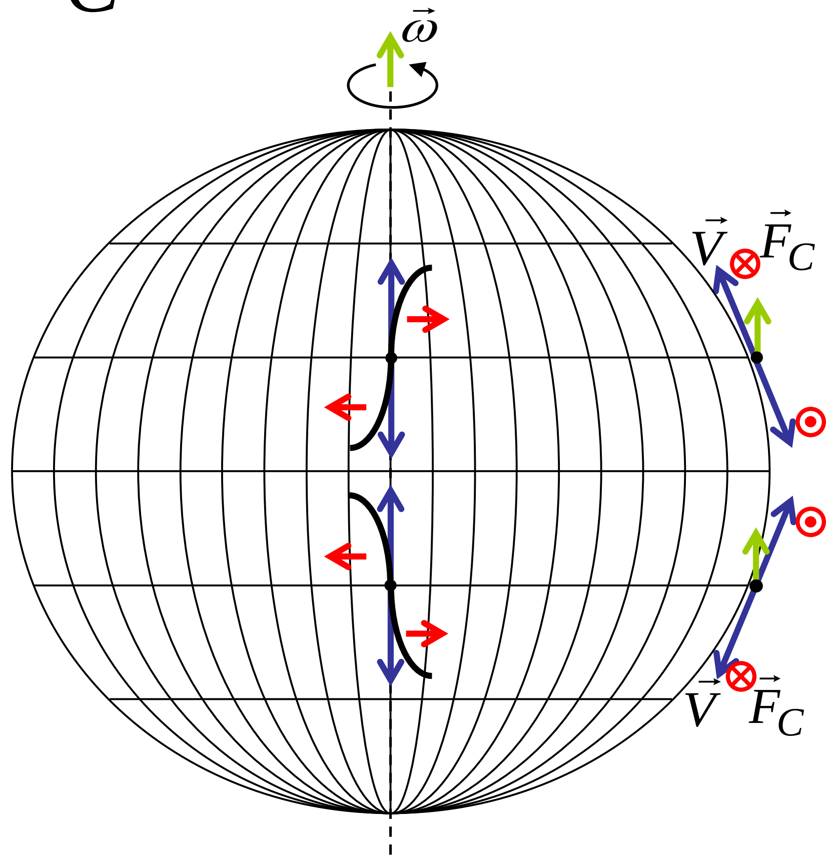
$$\vec{F}_C = -m \cdot 2\vec{\omega} \times \vec{V}$$

LINEAR MOTION IN A ROTATING REFERENCE FRAME



$$\vec{F}_C = -m \cdot 2\vec{\omega} \times \vec{V}$$

$$\vec{F}_C = -m \cdot 2\vec{\omega} \times \vec{V}$$

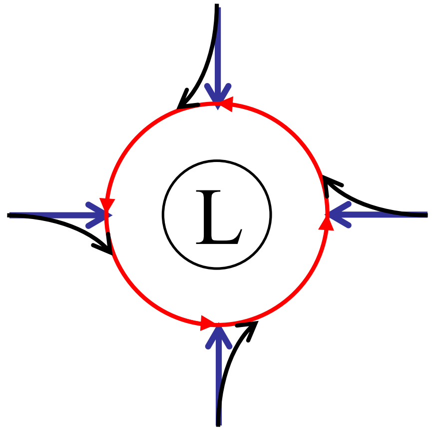


Coriolis pseudo force deflect moving bodies:

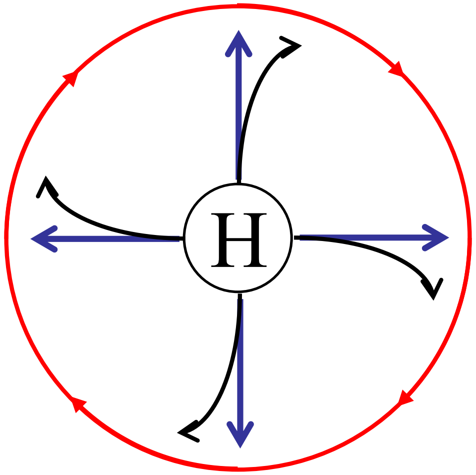
- in the northern hemisphere to the right

- in the southern hemisphere to the left

The winds blow to the low



The winds blow from the high



On the northern hemisphere:

The wind circulation near the low has the counter clockwise direction

The wind circulation near the high has the clockwise direction

FOUCAULT PENDULUM (1851)

$m = 28\ \mathrm{kg}$
$L = 67\ \mathrm{m}$

$$T = \frac{24}{\sin(\mathrm{latitude})}$$

Szczecin:Cathedral Basilica of St. James the Apostle

NEWTON'S SECOND LAW
IN NON-INERTIAL REFERENCE FRAME

$$m\vec{a}' = \vec{F}_w' = \vec{F}_w + \left(\vec{F}_i + \vec{F}_c + \vec{F}_C + \vec{F}_E\right)$$

$\vec{F}_w$ - net force of all real forces

$\vec{F}_i = -m\vec{a}_u$ - inertial pseudo force

$\vec{F}_c = -m \cdot \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$ - centrifugal pseudo force

$\vec{F}_C = -m \cdot 2\vec{\omega} \times \vec{V}'$ - Coriolis pseudo force

$\vec{F}_E = -m \cdot \vec{\varepsilon} \times \vec{r}$ - Euler pseudo force