Dynamics

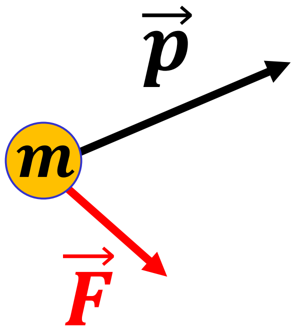
Dynamics is the branch of physics concerned with the study of forces and torques and their effect on motion.

The basic quantities

1) mass – m (bring about the **Inertia** – the resistance of any physical object to any change in its state of motion)

2) momentum – $\vec{p} = m\vec{V}$ (describe the state of motion)

3) force – $\vec{F}$ (the quantity changing the state of motion)



Newton's First Law

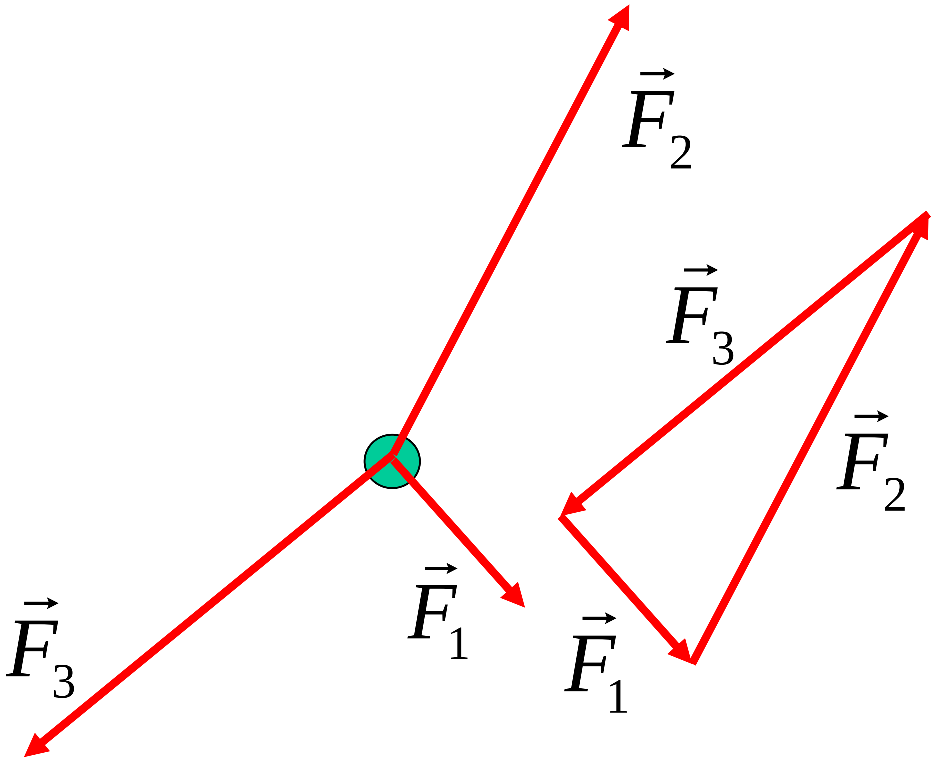
The state of motion remain constant

(the body remains at rest or continues to move at a constant velocity)

if the net force is equal zero.

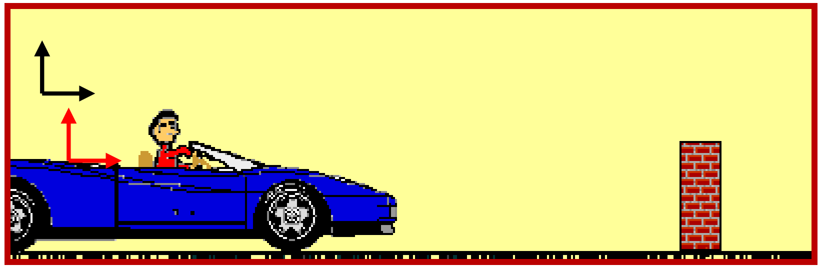
$$\vec{p} = \overrightarrow{\mathrm{const}} \iff \vec{F}_w = \vec{0}$$

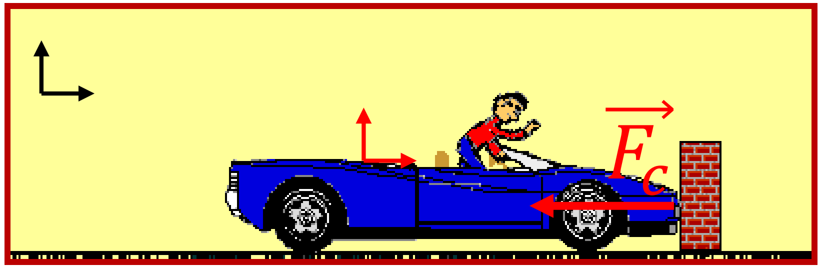
$$(\vec{a} = \vec{0})$$



Newton's First Law

Exist at least one frame of reference called an **inertial reference frame**, relative to which the motion of a particle not subject to net force is a straight line at a constant speed or rest.





$$\overrightarrow{V_c} = \overrightarrow{V_0} \qquad \overrightarrow{V_c} = \vec{0}$$
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Newton's Second Law

The rate of change of the momentum

is equal to the net force acting on the particle.

$$\vec{F}_w = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$\vec{F}_w = \frac{d\vec{p}}{dt} = \frac{d(m\vec{V})}{dt} = \frac{dm}{dt}\vec{V} + m\frac{d\vec{V}}{dt} = m\vec{a}$$

$$\frac{d\vec{V}}{dt} = \vec{a}$$ – the rate of change of the velocity

$$\frac{dm}{dt}$$ – the rate of change of the mass

In the majority of nonrelativistic cases the mass remain constant, and then: $$\frac{dm}{dt} = 0$$

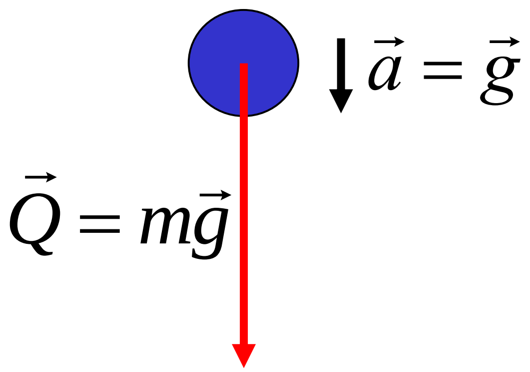
Newton’s Second Law

The rate of change of the momentum

is equal to the net force acting on the partcle.

$$\vec{F}_w = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \underset{m=\text{const}}{=} m\vec{a}$$

$$\vec{a} = \frac{1}{m} \cdot \vec{F}_w$$



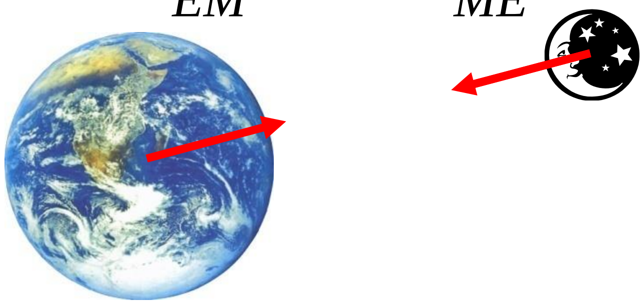
$$\left.\begin{array}{l} \vec{F}_w = \vec{Q} = m\vec{g} \\ \vec{F}_w = m\vec{a} \end{array}\right\} \Rightarrow \vec{a} = \vec{g}$$

Newton's Third Law

Forces are caused by the interactions of pair of bodies.

The force exerted by body B upon body A has the same magnitude and is in the same straight line but acts in opposite direction to the force exerted by body A upon body B.

$$\vec{F}_{EM} = -\vec{F}_{ME}$$



$$\vec{F}_{AB} = -\vec{F}_{BA}$$

FORCES AT MECHANICS

Gravitation

Newton's law of universal gravitation (1687):

Every particle attracts every other particle

with a force $\vec{F}$

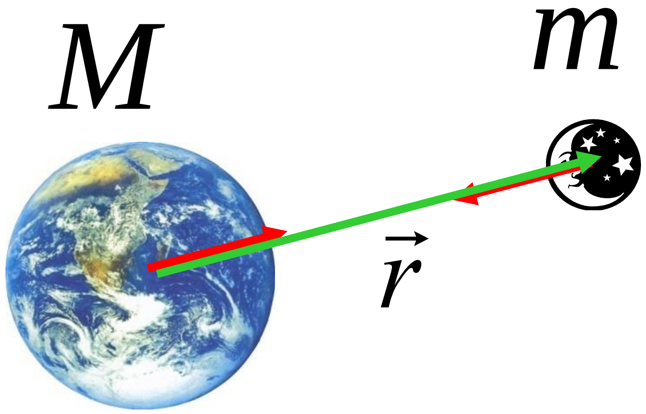
proportional

to the product of the masses M and m

and inversely proportional to the square of the distance between them.

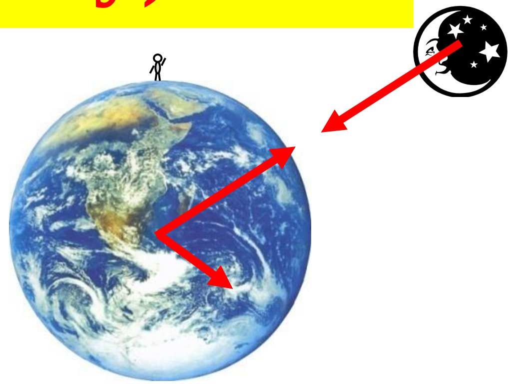
$$\vec{F} = -G\frac{M \cdot m}{r^2}\frac{\vec{r}}{r}$$

$$G = 6{,}67 \cdot 10^{-11}\frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2}$$

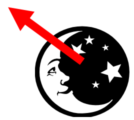


Gravitational acceleration (standard gravity)

$$\vec{F} = -G\frac{Mm}{r^2}\frac{\vec{r}}{r}$$

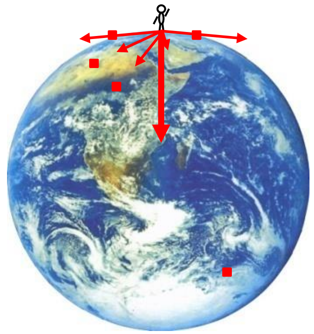


$$\left.\begin{aligned} r \approx R_E \Rightarrow F = G\frac{M}{R_E^2}m \\ Q = mg \end{aligned}\right\} \Rightarrow g = G\frac{M}{R_E^2}$$



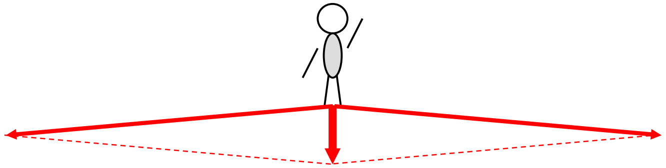
$$\left.\begin{aligned} G &= 6{,}67408(31)\cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg}\cdot \mathrm{s}^2} \\ M_E &\approx 5{,}97\cdot 10^{24}\ \mathrm{kg} \\ \mathrm{R_E} &\approx 6378\ \mathrm{km} \end{aligned}\right\} \Rightarrow g = 9{,}79\ \mathrm{m/s^2}$$

$$F = -G\frac{Mm}{R_E^2} \qquad \vec{F}_i = -G\frac{\mathrm{d}M\cdot m}{r_i^{\,2}}\frac{\vec{r}_i}{r_i}$$



According to the principle of superposition:

$$\vec{F} = \sum \vec{F}_i = \sum -G\frac{\mathrm{d}M\cdot m}{r_i^{\,2}}\frac{\vec{r}_i}{r_i}$$



$$\vec{F} = \int \mathrm{d}\vec{F} = \int -G\frac{m}{r_i^{\,2}}\frac{\vec{r}_i}{r_i}\,\mathrm{d}M$$

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its centre.

Gravitation ⊥ Earth

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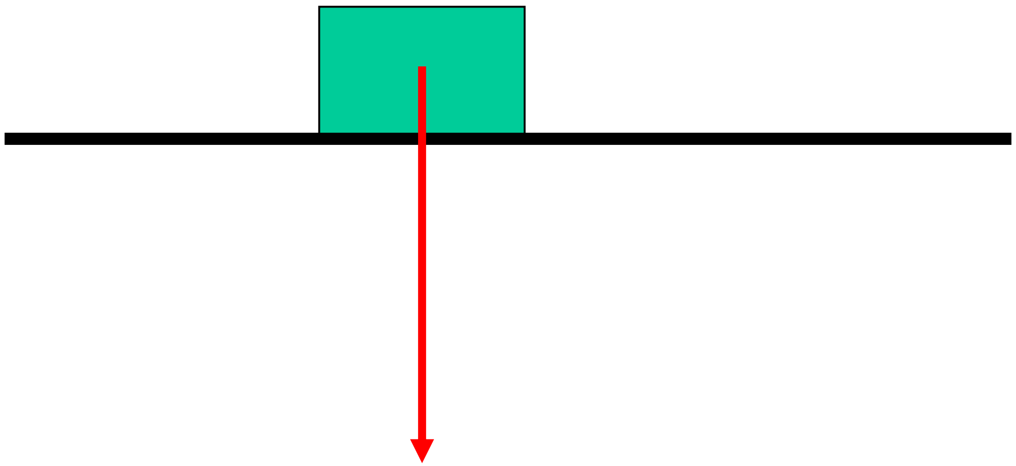
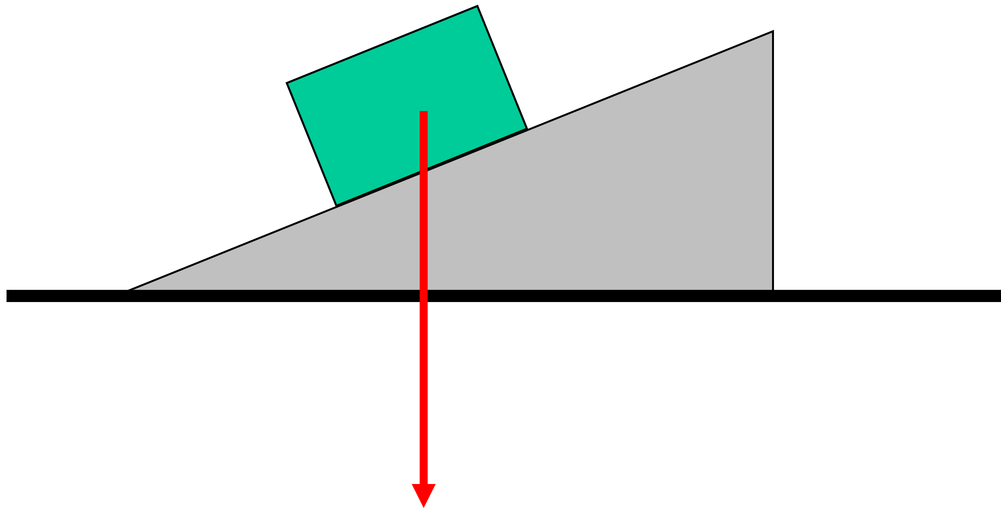
Gravitation ⊥ Earth



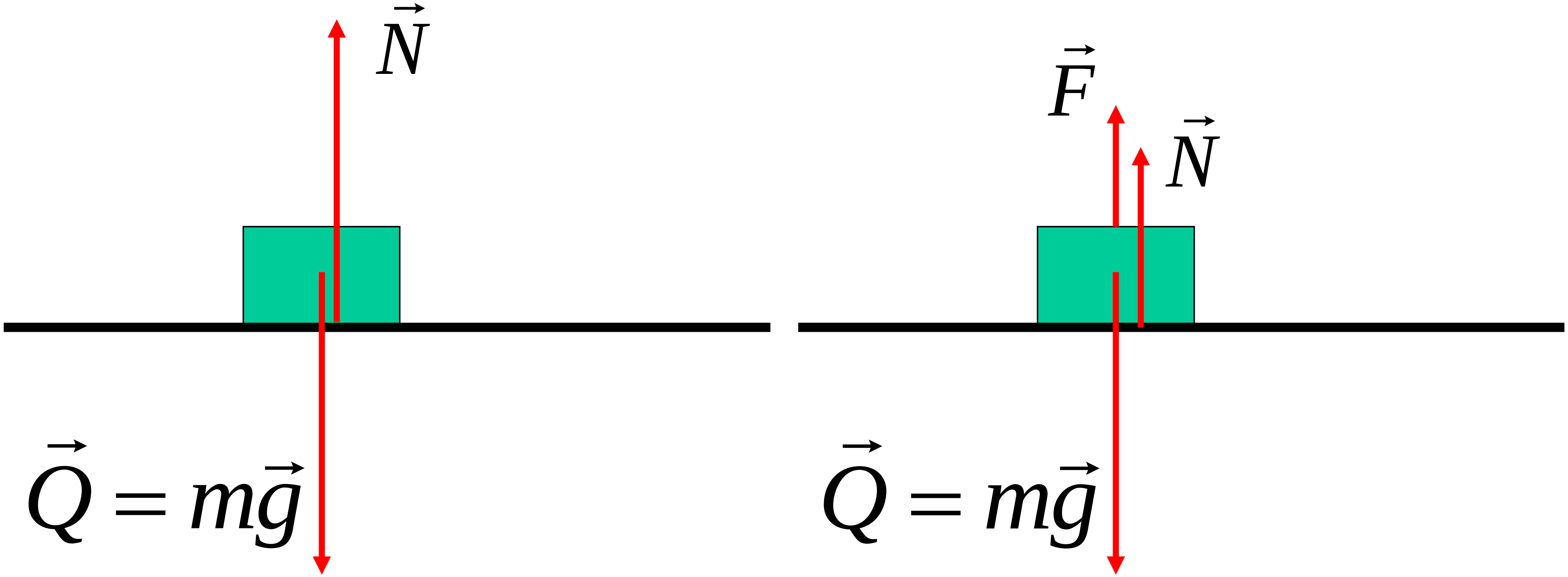
$$\vec{Q} = m\vec{g}$$



$$\vec{Q} = m\vec{g}$$

Normal force

Normal force ⊥ surface



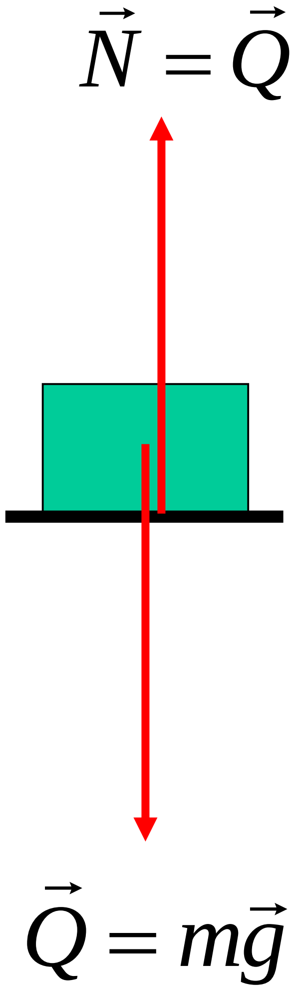
$$\vec{N} + \vec{Q} = \vec{0} \Rightarrow \vec{N} = -\vec{Q}$$

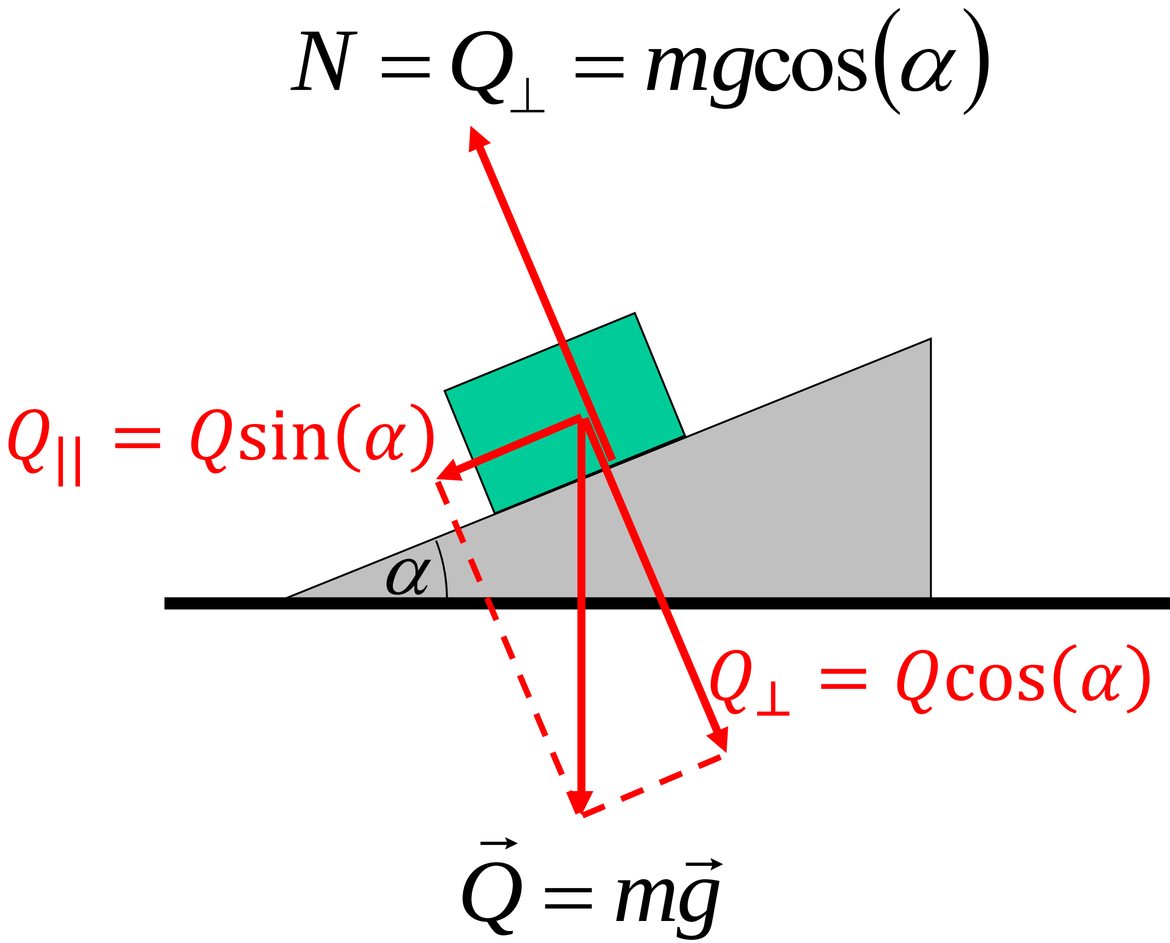
$$N = mg$$

$$\vec{N} + \vec{Q} + \vec{F} = \vec{0} \Rightarrow \vec{N} = -\vec{Q} - \vec{F}$$

$$N = mg - F$$

$$N < mg \;\; !!!!$$

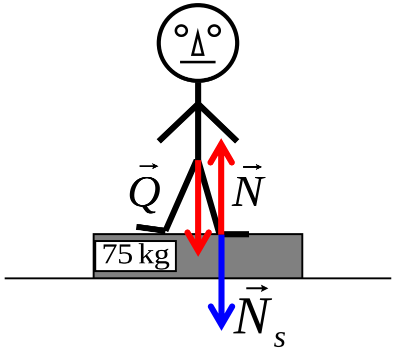




$Q_{\perp}$- press the body to the inclined plane

$Q_{||}$- slip down the body from the inclined plane

Body on a scale



$$m = 75\text{kg}$$

$$\vec{Q} \Rightarrow \vec{N}_s \Rightarrow \vec{N} = -\vec{N}_s$$

$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} = \vec{0}$$

$$-Q + N = 0$$

$$N = Q$$

$$N = mg$$

$$N_s = mg$$

$$\frac{N_s}{g} = m$$

A weighting scale indicates the force applied, calibrated in kilograms!!!!!

Static friction

$$\vec{T}_s = -\vec{F}$$

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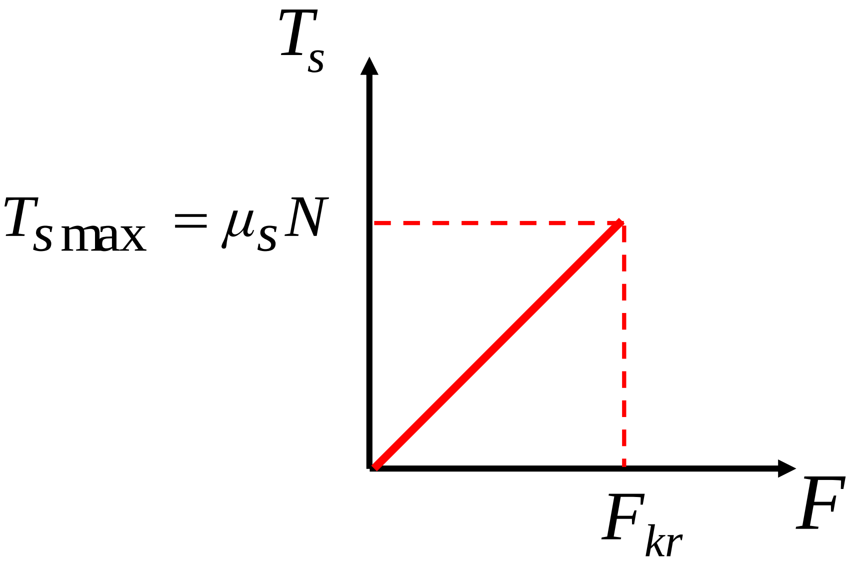
$$T_{s\max} = F_{kr} = \mu_s N$$

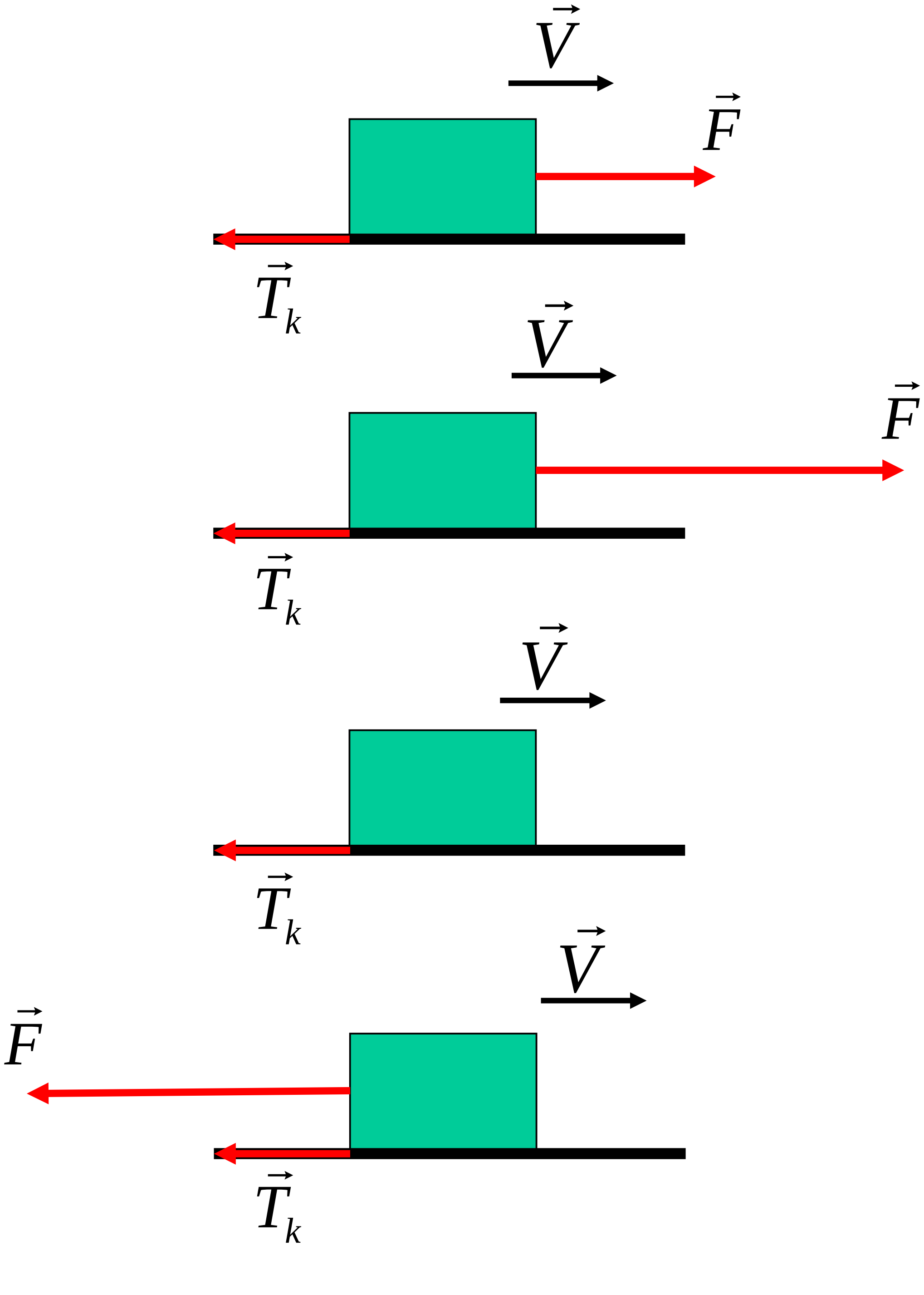
$$F_{kr} = \mu_s N$$

$$T_s \le \mu_s N$$



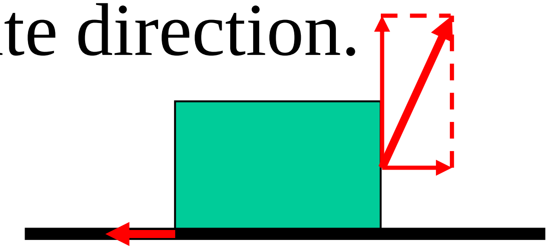
Kinetic friction

$$T_k = \mu_k N$$



Properties of friction

1) If the body does not move relative to the surface, then the static frictional force $\vec{T}_s$ is equal in magnitude to the parallel to the surface component of total force $\vec{F}$, but has opposite direction.



2) The maximum magnitude of static friction $\vec{T}_s$ is:

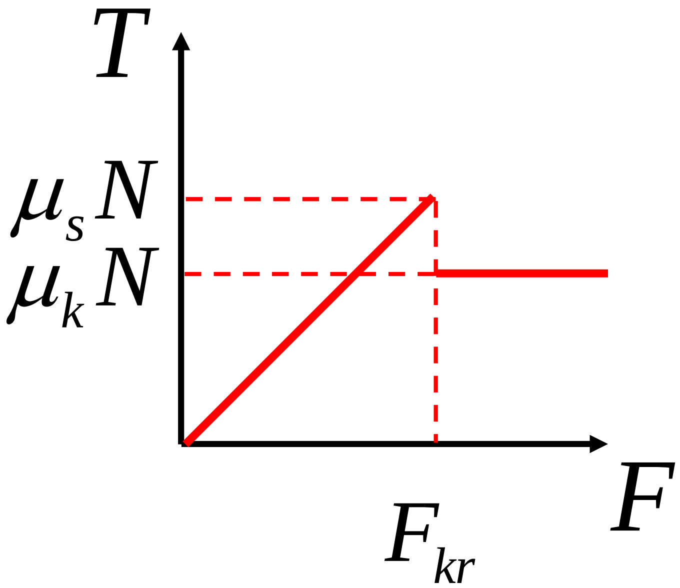
$$T_{s\max} = \mu_s N$$

where μ_s is the coefficient of static friction N is the magnitude of the normal force.

3) If the body slide along the surface, then the kinetic friction $\vec{T}_k$ has the direction opposite to the velocity and the magnitude

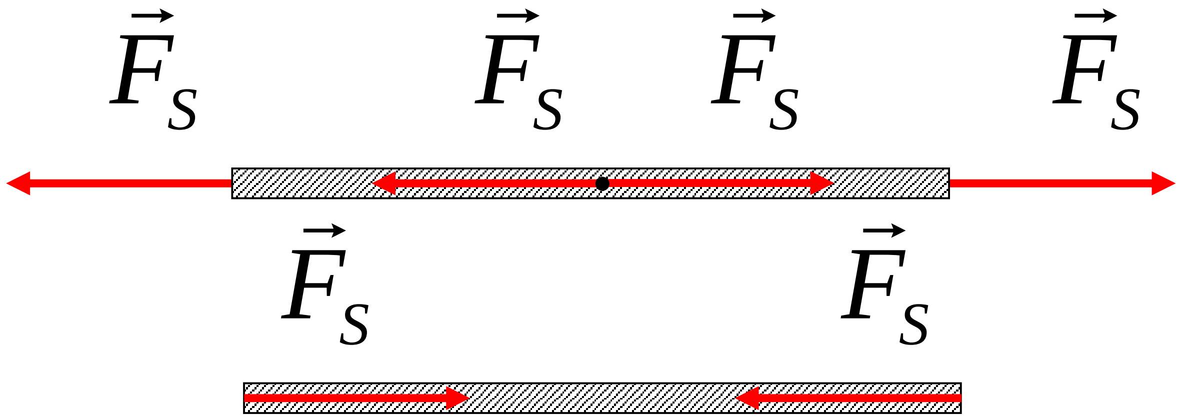
$$T_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force.



$$\mu_s > \mu_k$$

Stress

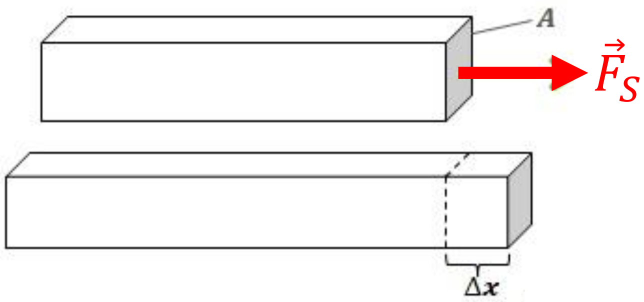


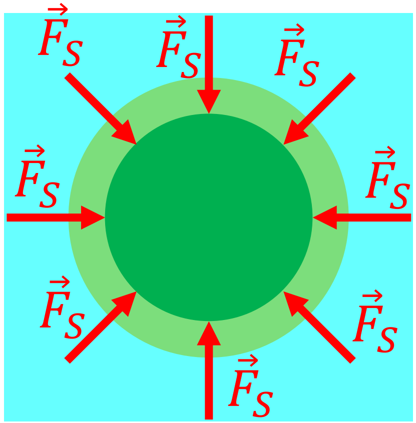
Stress – a system of forces in equilibrium producing in a body:

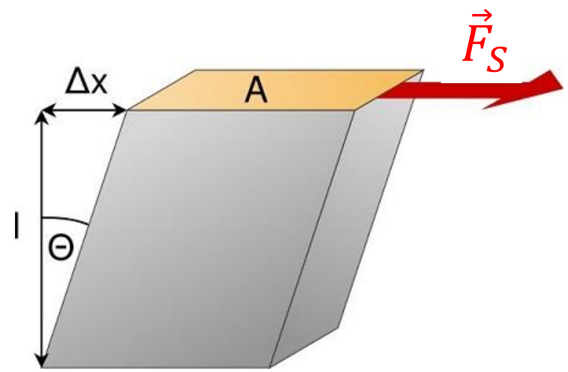
– tensile stress → linear strain: the change in length per unit length

– hydraulic stress → volume (bulk) strain: the change in volume per unit volume

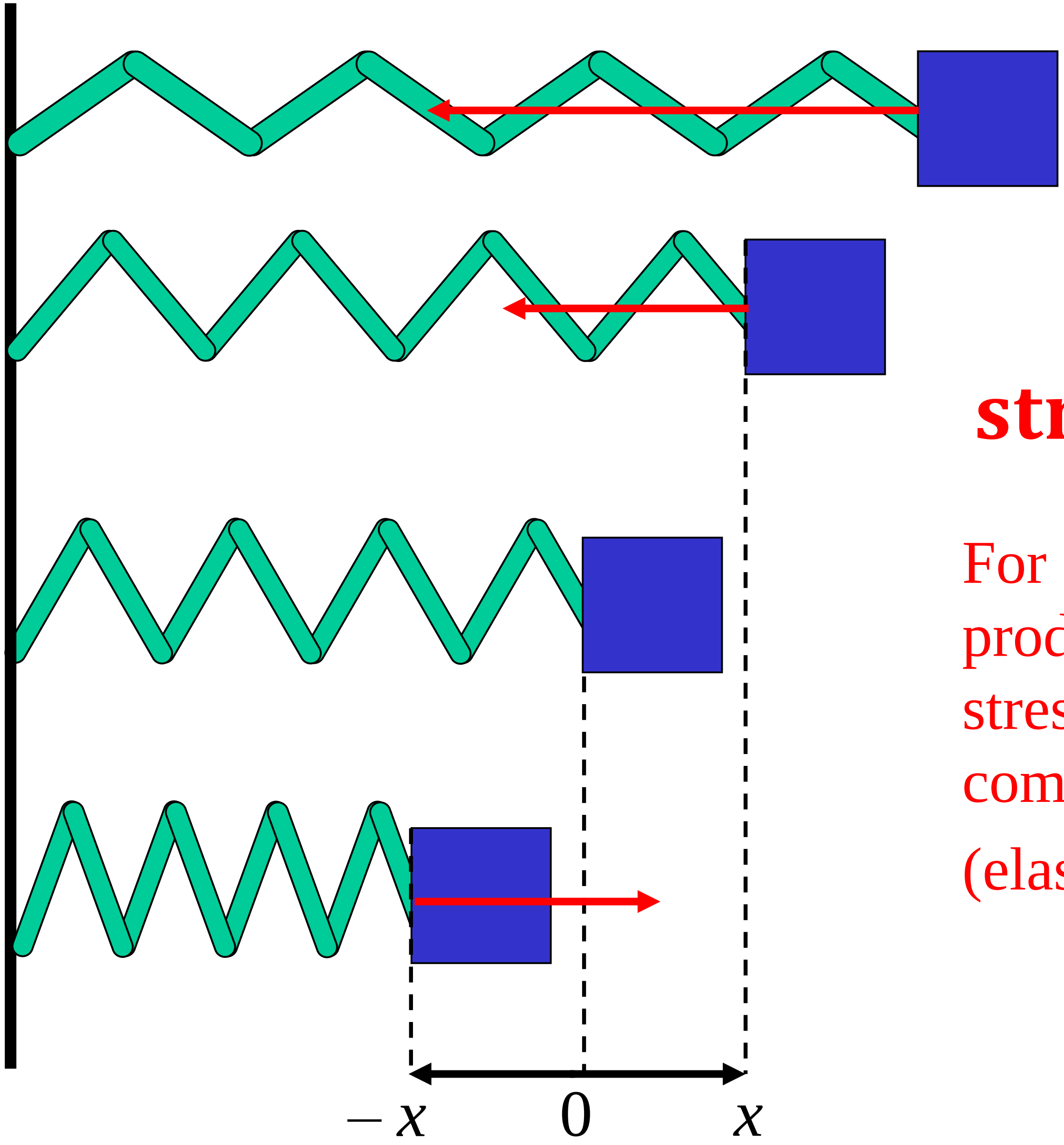
– shearing stress → shear strain: angular deformation without change in volume







Hooke's law (1676)



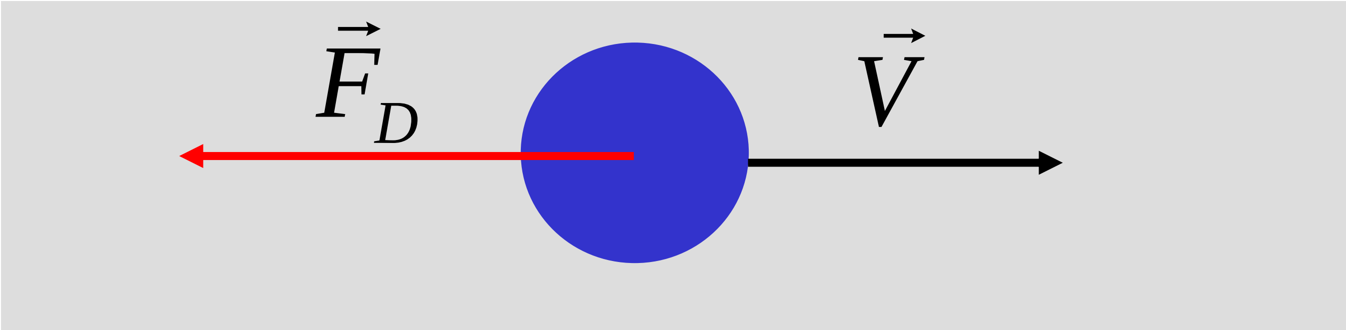
$$\vec{F}_s = -k \cdot \vec{x}$$

stress = modulus · **strain**

For a certain range of stresses, the produced strain is proportional to the stress applied, and disappears completely on removal of the stress.

(elastic limit, spring force)

Drag force



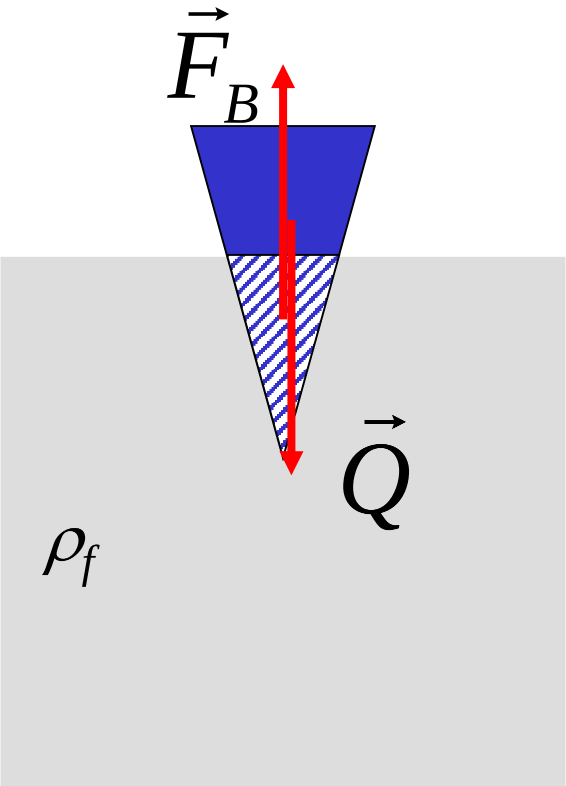
When a body and a fluid (gas or liquid) are in relative motion, the body experiences a drag force, parallel to the direction of relative motion but in opposite direction and magnitude:

- at low speed: $$\vec{F}_D = -bV\frac{\vec{V}}{V}$$
- at higher speed: $$\vec{F}_D = -\frac{1}{2}C\rho AV^2\frac{\vec{V}}{V}$$

C – the drag coefficient (0.4 ÷ 1)
ρ – the fluid density
A – the effective cross-section area of the body
V – the relative velocity

Buoyant force

Buoyancy – the tendency of a fluid (gas of liquid) to exert a lifting forces on a body wholly or partly immersed in it.



$$\vec{F}_B = -\rho_f V_d \vec{g}$$

$$\rho = \frac{m}{V} \quad \Rightarrow \rho V = m$$

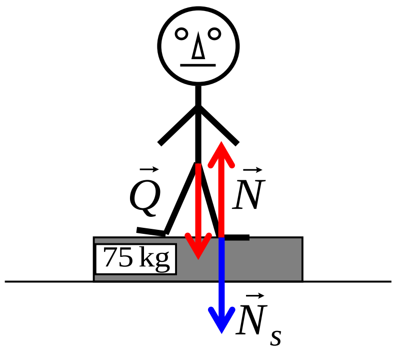
$$\rho_f V_d = m_{fd}$$

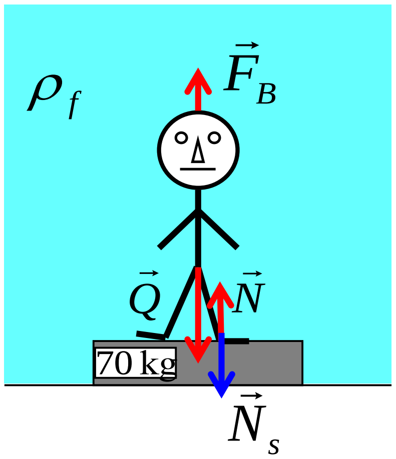
↑ the mass of the fluid that the body displaced

$$\vec{F}_B = -m_{fd} \vec{g}$$

The weight of an object act effectively at the object's centre of mas.

The buoyant force act effectively at a point called the centre of buoyancy.





$$\vec{a} = \vec{0} \Rightarrow \vec{F}_w = \vec{0}$$

$$\vec{Q} + \vec{N} + \vec{F}_B = \vec{0}$$

$$-Q + N + F_B = 0$$

$$N = Q - F_B$$

$$N_s = N = mg - \rho_f V_d g$$

$$N_s = Q = mg$$

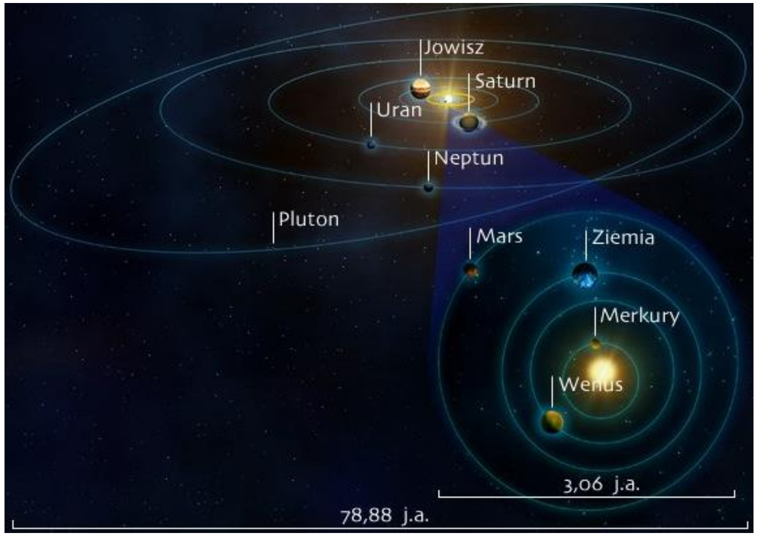
$$N_s = mg - m_{fd} g$$

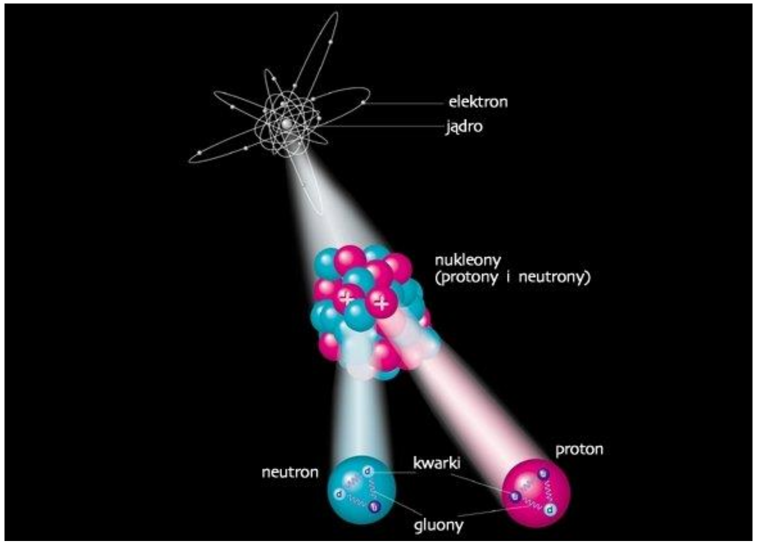
$$N_s = mg - Q_{fd}$$

A body fully or partially immersed in a fluid is buoyed up by e force equal to the weight of the fluid that the body displaced.
(Archimedes' principle)

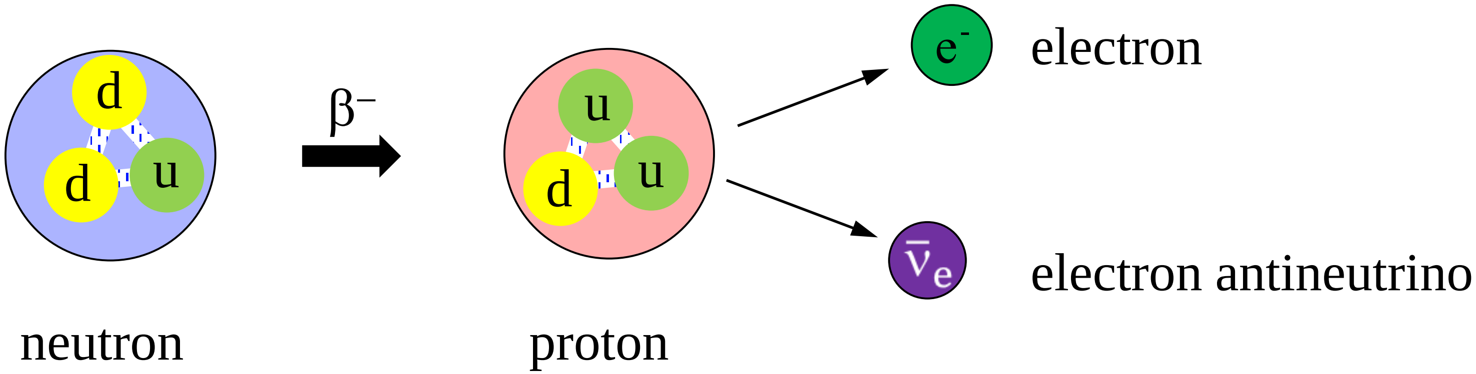
Fundamental interaction

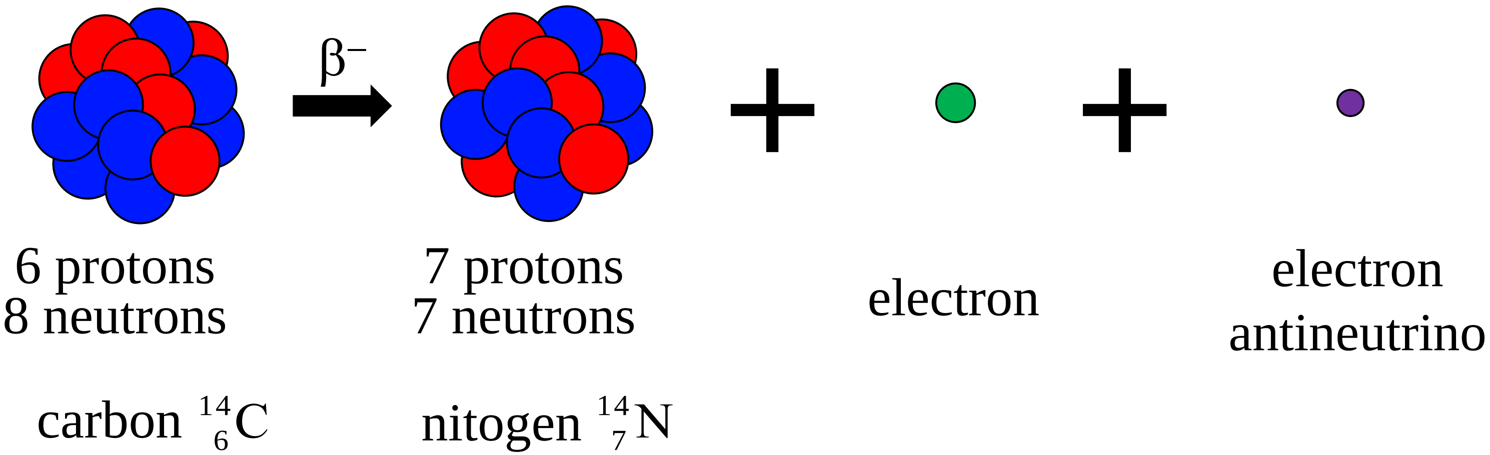
Interaction	Relative strengthh	Range	Particle experiencing	Example
Gravitation	10^{-38}	∞	massive particles	Solar System
Weak	10^{-14}	10^{-18}	gauge bosons, leptons	β decay
Electromagnetic	10^{-2}	∞	electrically charged	chemical bonds, atoms
Strong	1	10^{-15}	quarks, hadrons (p, n)	atomic nucleus





β^- decay


d
d
u
β^-
u
d
u
e^-
electron
$\bar{\nu}_e$
electron antineutrino
neutron
proton


β^-
+
+
6 protons
8 neutrons
7 protons
7 neutrons
electron
electron
antineutrino
carbon $^{14}_{6}C$
nitogen $^{14}_{7}N$

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