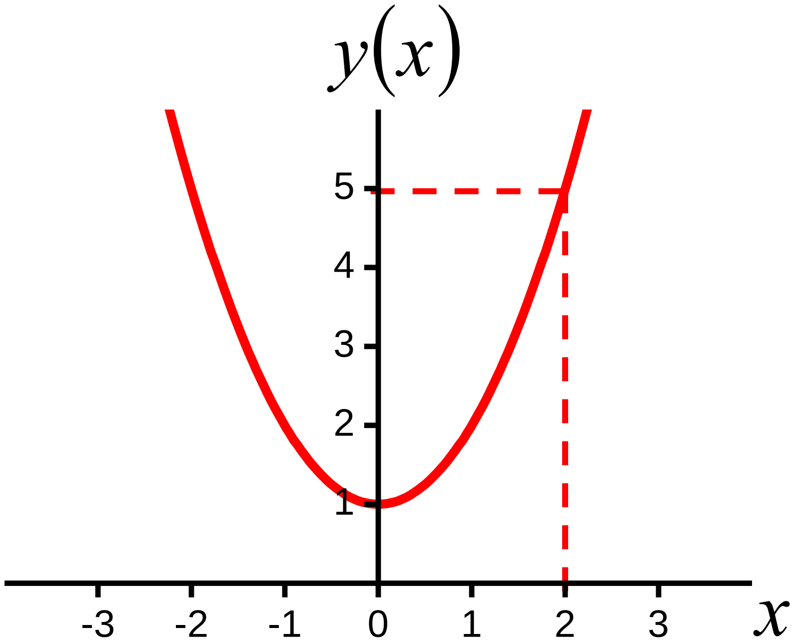
Function

– a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output

$$y(x) = x^2 + 1$$



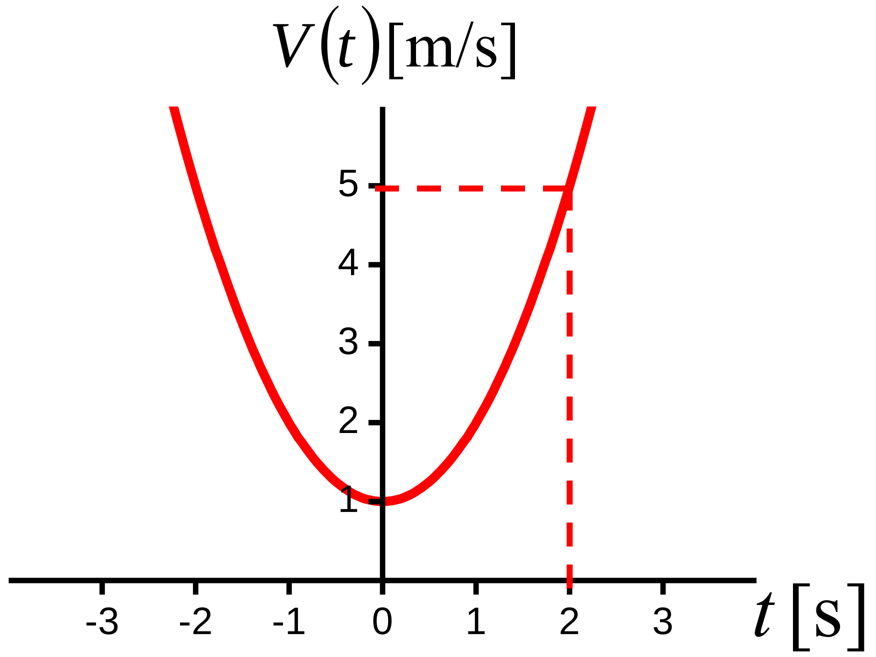
x – independent variable – argument of function

y – dependent variable – function value

$$x = 2 \Rightarrow y(2) = 5$$

Function

$$V(t) = t^2 + 1$$



t – time – independent variable – function argument

V – velocity – dependent variable – function value

$$t = 2\,\text{s} \Rightarrow V(2) = 5\,\text{m/s}$$

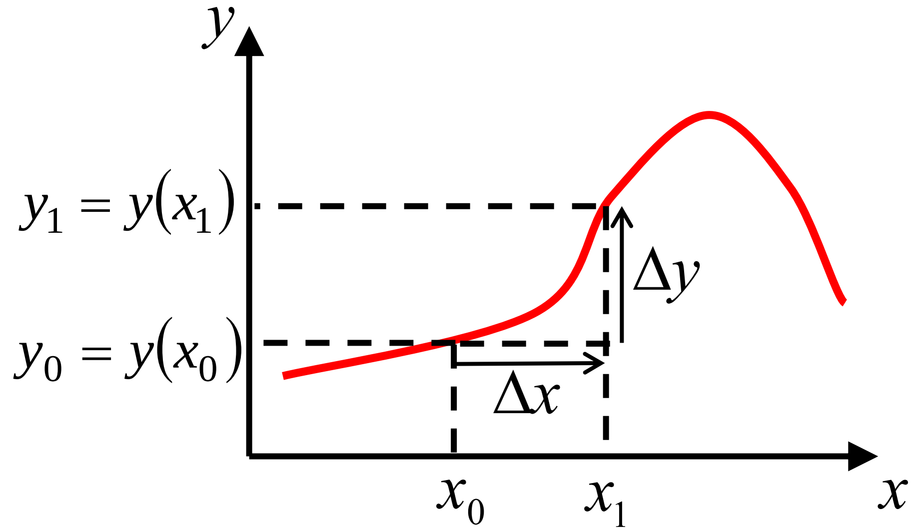
Change in variable value

The increment in any variable is marked with a symbol „Δ”

and is the difference between the new and the old value.

$$\Delta x = x_1 - x_0$$

$$\Delta y = y_1 - y_0$$



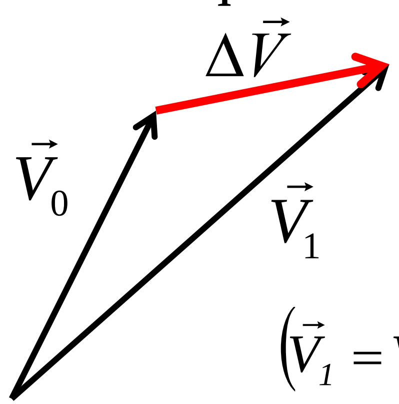
The increment could be positive (function increases) as well as negative (function decreases)

The increment could be calculated for scalar and vector quantities:

$\Delta t = t_1 - t_0$: the increment of the time

$\Delta E = E_1 - E_0$: the increment of the energy

$\Delta \vec{V} = \vec{V}_1 - \vec{V}_0$: the increment of the velocity vector



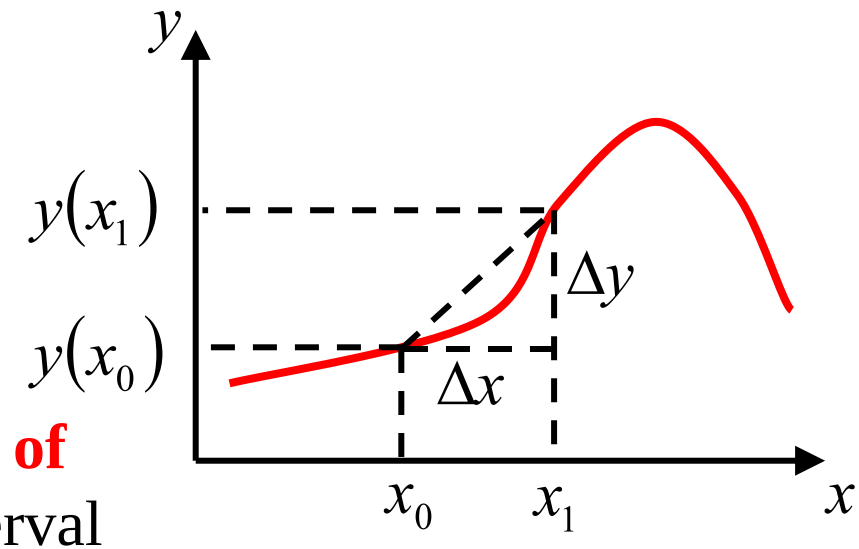
$\left(\vec{V}_1 = \vec{V}_0 + \Delta\vec{V}\right)$

Difference quotient

Quotient

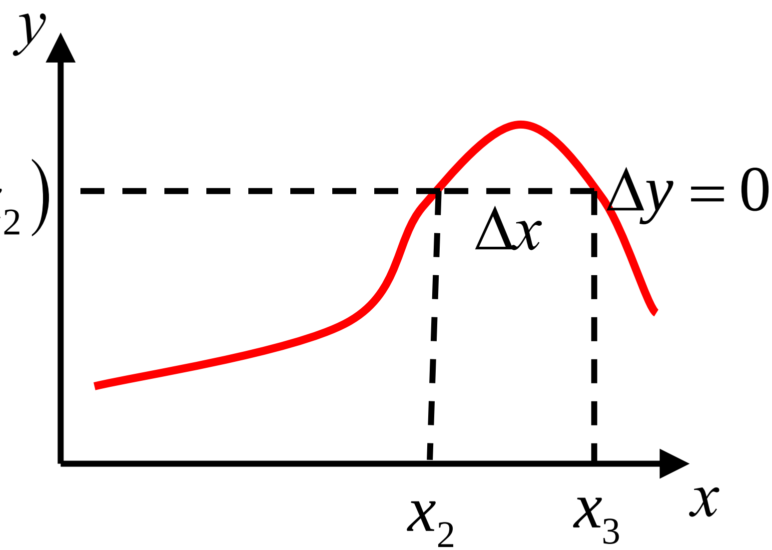
$$\frac{\Delta y}{\Delta x} = \frac{y(x_1) - y(x_0)}{x_1 - x_0}$$

is called difference quotient.

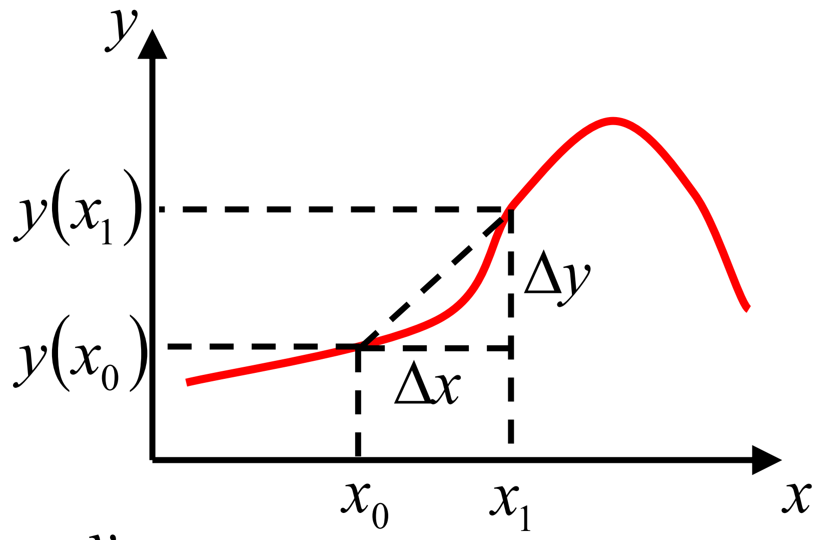


It is a measure of **the average rate of change** of the function over an interval from x_0 to x_1.

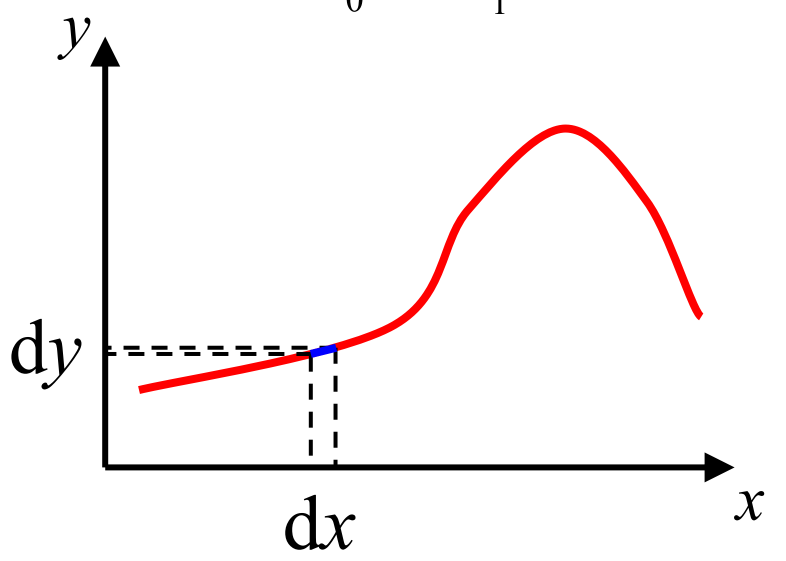
$$\frac{\Delta y}{\Delta x} = 0 \Leftarrow y(x_3) = y(x_2)$$



Difference quotient



To determine the actual rate of change of y (e.g. at a particular time or space point), the difference quotient has to be calculated for the change x by an infinitely small value: $\Delta x \to 0$.



To distinguish the infinitely small increment (approach zero) from the finite one, the letter „Δ" is replaced by the letter „d":

$$\Delta x \Longrightarrow \mathrm{d}x \qquad \Delta y \Longrightarrow \mathrm{d}y$$

so

$$\frac{\Delta y}{\Delta x} \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x}$$

Derivative

The limit of the difference quotient for infinitesimal small Δx

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y(x_0 + \Delta x) - y(x_0)}{\Delta x}$$

is defined to be **the derivative of the function y at x_0**,
and written as:

$$\frac{\mathrm{d}y}{\mathrm{d}x} \quad \text{or} \quad y'(x_0).$$

Derivative

At mathematics:

The derivative $\frac{dy}{dx}$ is **the ratio of the change** (instantaneous) in the dependent variable y to that of the independent variable x .

At physics:

The derivative $\frac{dy}{dx}$ is **the ratio of the change** (instantaneous) in physical quantity y to that of the physical quantity x .

e.g.:

$\frac{d\vec{V}}{dt}$ - is **the ratio of the change** of the velocity vector with respect to the time

$\frac{d\vec{p}}{dt}$ - is **the ratio of the change** of the momentum vector with respect to the time

$\frac{dE_p}{dx}$ - is **the ratio of the change** of the potential energy with respect to the position

Derivative

$\frac{\mathrm{d}\vec{V}}{\mathrm{d}t}$ – is read as:

„the derivative of the velocity with respect to the time”

or

„di vi by di ti”

or

„di vi over di ti”

Inverse function

$$y = x^2 \quad \Rightarrow \quad x = \pm\sqrt{y}$$

$$y = \frac{1}{2}x^2 - 1 \quad \Rightarrow \quad x = \pm\sqrt{2(y+1)}$$

$$y = \sin(x) \quad \Rightarrow \quad x = \arcsin(y)$$

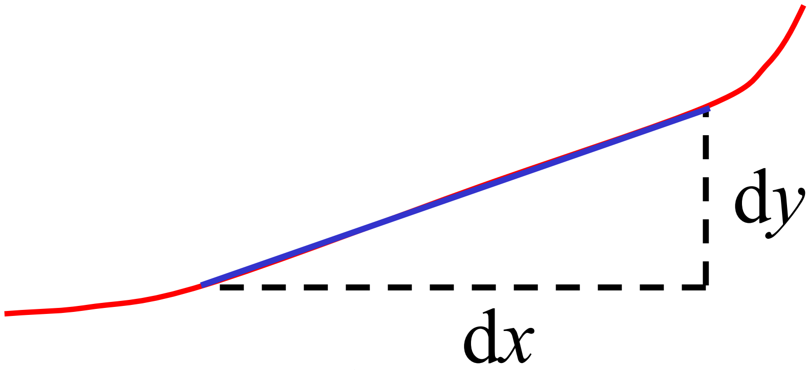
$$y = \ln(x) \quad \Rightarrow \quad x = e^y$$

$$\frac{dy}{dx} = f(x) \quad \Rightarrow \quad ?$$

Integral

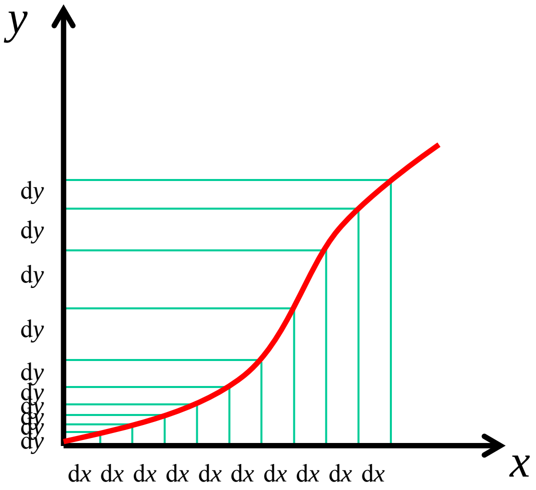
$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x) \,/\cdot \mathrm{d}x$$

$$\mathrm{d}y = f(x)\mathrm{d}x$$



All increments could be added:

$$\sum \mathrm{d}y = \sum f(x)\mathrm{d}x$$



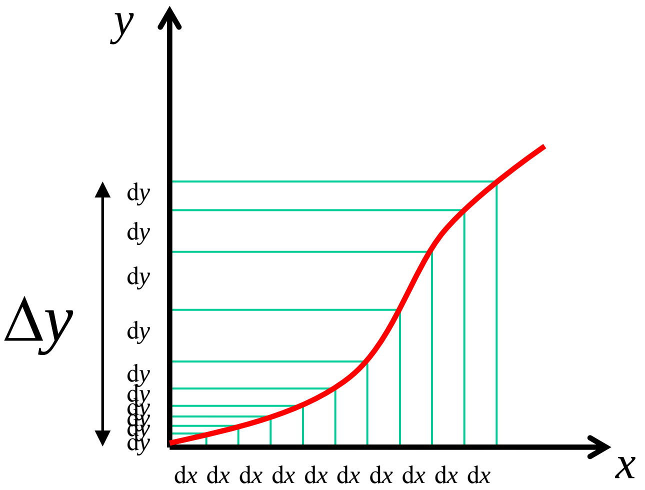
An infinite sum of infinitesimal small quantities is called the integral and marked by symbol $\int$

$$\int \mathrm{d}y = \int f(x)\mathrm{d}x$$

Integral

$$\int dy = \int f(x)dx$$

$$\Delta y = \int f(x)dx$$



$$\Delta y = y_1 - y_0 \Rightarrow y_1 = y_0 + \Delta y$$

$$y_1 = y_0 + \int_{x_0}^{x_1} f(x)dx$$ – the definite integral

$$y(x) = \int f(x)dx$$ – the indefinite integral

$\int f(x)dx$ – is read as: „the integral of f over x"
or „ the x integral of the f"

Integral = Antiderivative

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x) \quad \Leftrightarrow y(x) = \int f(x)\mathrm{d}x$$

The integral is the inverse operation to the derivative

– the antiderivative.

So:

If $f(x)$ is a derivative of y over x

then y is an integral of f over x.

E.g.:

$$\vec{V} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \quad \Leftrightarrow \vec{r} = \int \vec{V}\mathrm{d}t$$

The velocity is the derivative of the position over the time

⇔ The position is the integral of the velocity over the time

(the time integral of the velocity)