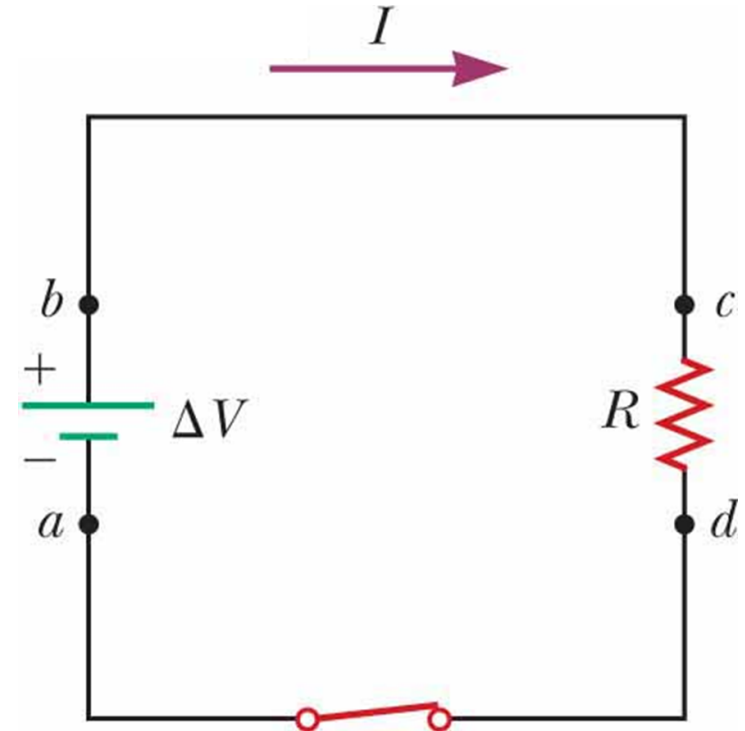


Direct Current Circuits



Physics for Scientists and Engineers, 10e
Raymond A. Serway
John W. Jewett, Jr.

Electromotive Force



- Battery – source of energy for circuits
- Battery: *source of electromotive force* or *source of emf*
 - *Electromotive force* unfortunate historical term describing potential difference in volts
 - **Emf** of battery: **maximum possible voltage battery can provide between its terminals**
- Think of source of emf as “charge pump”: when electric potential difference exists between two points – positive charges move “uphill” from lower potential to higher

Electromotive Force

$$U = \Delta V = V_d - V_a = \varepsilon - Ir$$

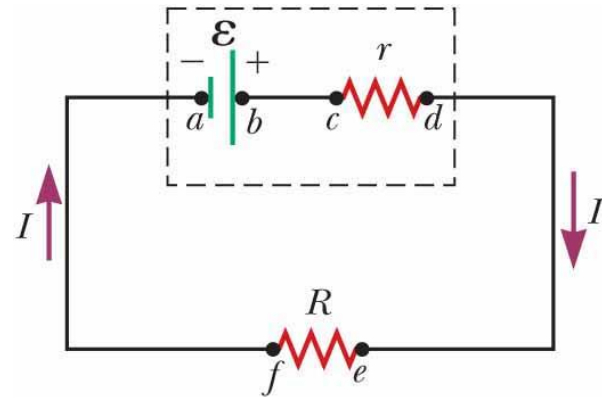
$$\varepsilon = IR + Ir$$

$$I = \frac{\varepsilon}{R + r}$$

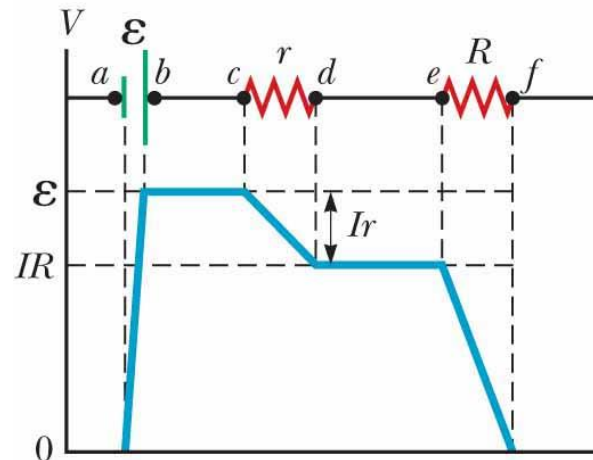
$$U = \varepsilon \left(1 - \frac{r}{R + r} \right)$$

$$U \approx \varepsilon \Leftrightarrow R \gg r$$

$(I \approx 0)$



a



b

Resistors in Series and Equivalent Resistance

Same amount of charge passes through both resistors in given time interval: Currents are same in both resistors.

$$I = I_1 = I_2$$

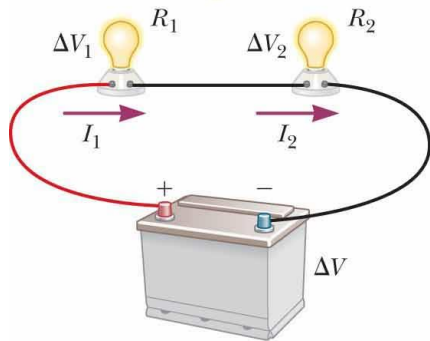
Potential difference applied across series combination of resistors divides between resistors

$$\Delta V = \Delta V_1 + \Delta V_2 = I_1 R_1 + I_2 R_2$$

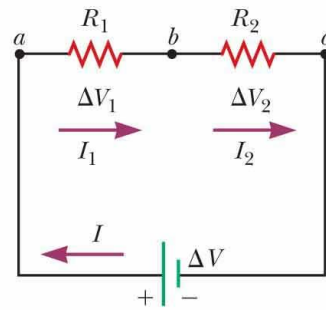
$$\Delta V = I R_{\text{eq}} = I (R_1 + R_2)$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \dots$$

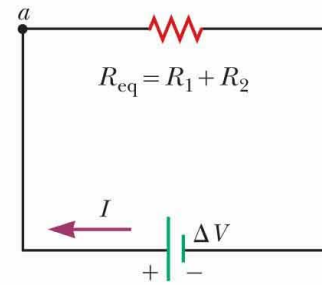
A pictorial representation of two resistors connected in series to a battery



A circuit diagram showing the two resistors connected in series to a battery



A circuit diagram showing the equivalent resistance of the resistors in series



a

b

c

Resistors in Parallel

$$\Delta V = \Delta V_1 = \Delta V_2$$

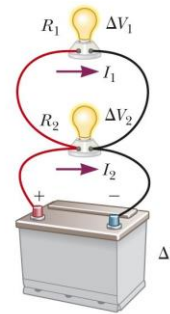
$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

$$I = \frac{\Delta V}{R_{\text{eq}}}$$

$$\frac{\Delta V}{R_{\text{eq}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow$$

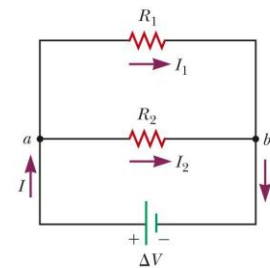
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

A pictorial representation of two resistors connected in parallel to a battery



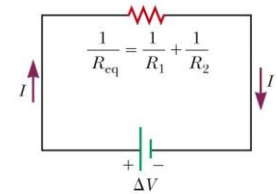
a

A circuit diagram showing the two resistors connected in parallel to a battery



b

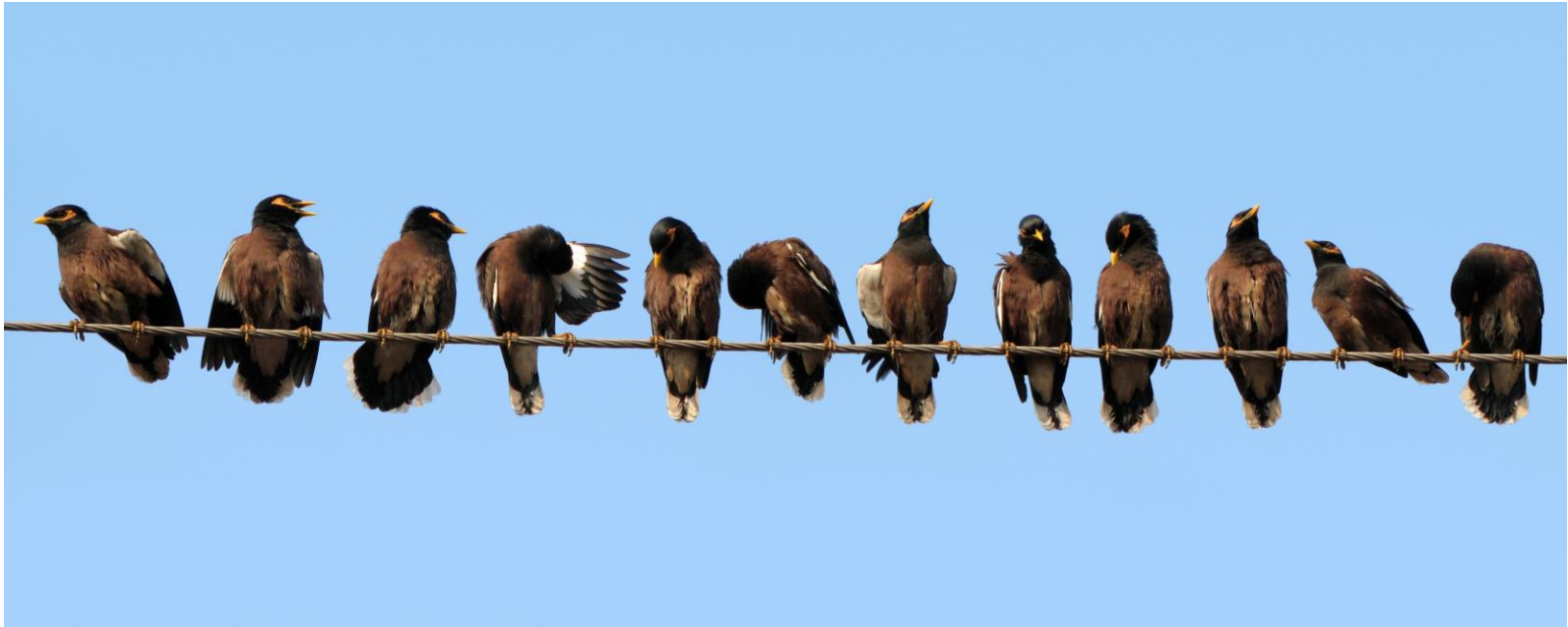
A circuit diagram showing the equivalent resistance of the resistors in parallel



c

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Birds on a Power Wire



- What about the bird on the wire?
 - It may be that wire insulated, keeping the bird safe.
- Even if wire not insulated → bird's feet closer together
- Wire most likely has smaller resistivity
- Both factors lead to very small potential difference between feet of bird when it is connected in parallel with wire
 - Very little current in bird's body

Kirchhoff's Rules

1. Junction rule. At any junction, the sum of the currents must equal zero:

$$\sum_{\text{junction}} I = 0$$

2. Loop rule. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0$$

Junction Rule

1. Junction rule. At any junction, the sum of the currents must equal zero:

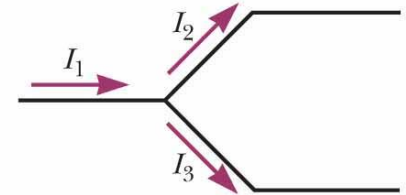
$$\sum_{\text{junction}} I = 0$$

$$I_1 - I_2 - I_3 = 0$$

Kirchhoff's first rule: conservation of electric charge

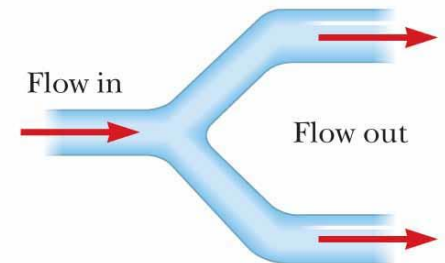
- All charges that enter given point in circuit must leave that point
- Charge cannot build up or disappear at a point.

The total amount of charge flowing in the branches on the right must equal the amount flowing in the single branch on the left.



a

The total amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left.

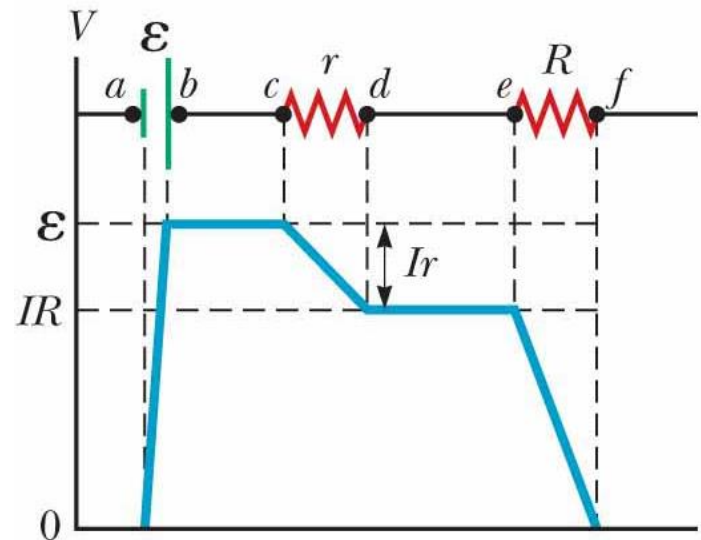


b

Loop Rule

2. Loop rule. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0$$



Kirchhoff's second rule:

- For conservative forces the change of potential energy is zero on closed path
- Sum of increases in potential in some circuit elements = sum of decreases as you pass through others

Sign Conventions for Kirchhoff's Rules

Water flows from higher point (higher gravitational potential) to lower one (lower gravitational potential).

Current flows from higher electric potential to lower electric potential.

$$V_a > V_b \rightarrow U = \Delta V = V_b - V_a < 0$$

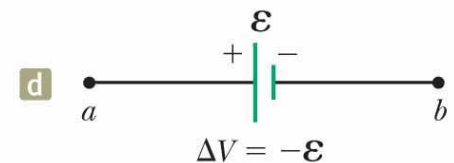
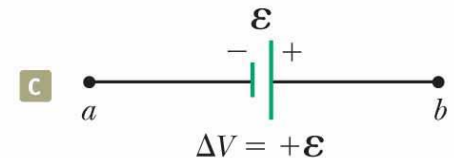
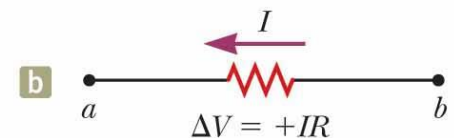
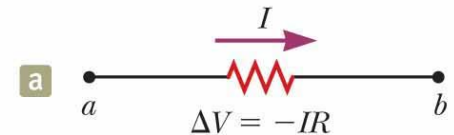
$$V_b > V_a \rightarrow U = \Delta V = V_b - V_a > 0$$

Source of emf: „-” – lower potential,
„+” – higher potential

$$V_b > V_a \rightarrow U = \Delta V = V_b - V_a > 0$$

$$V_a > V_b \rightarrow U = \Delta V = V_b - V_a < 0$$

In each diagram, $\Delta V = V_b - V_a$ and the circuit element is traversed from a to b , left to right.



Problem-Solving Strategy: Kirchhoff's Rules

1. Conceptualize
2. Categorize
3. Analyze
4. Finalize

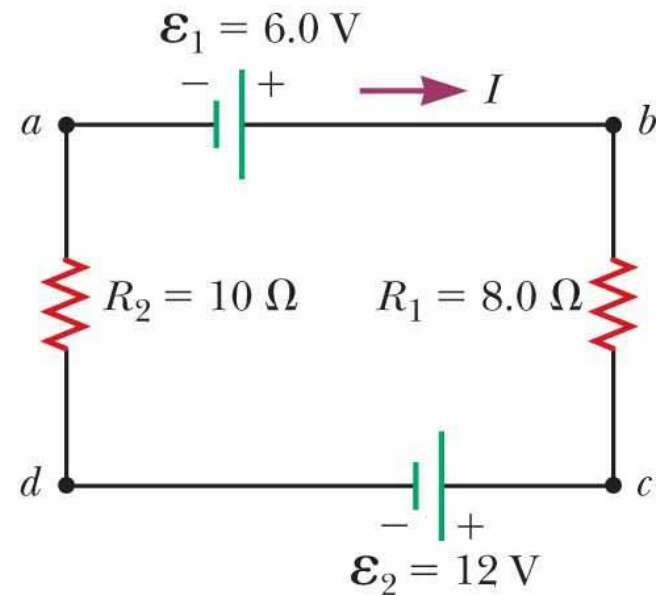
Example 27.6: A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries as shown in the figure. (Neglect the internal resistances of the batteries.) Find the current in the circuit.

$$\sum \Delta V = 0 \Rightarrow \varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

$$\begin{aligned} I &= \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} \\ &= \frac{6.0 \text{ V} - 12 \text{ V}}{8.0 \Omega + 10 \Omega} = \boxed{-0.33 \text{ A}} \end{aligned}$$

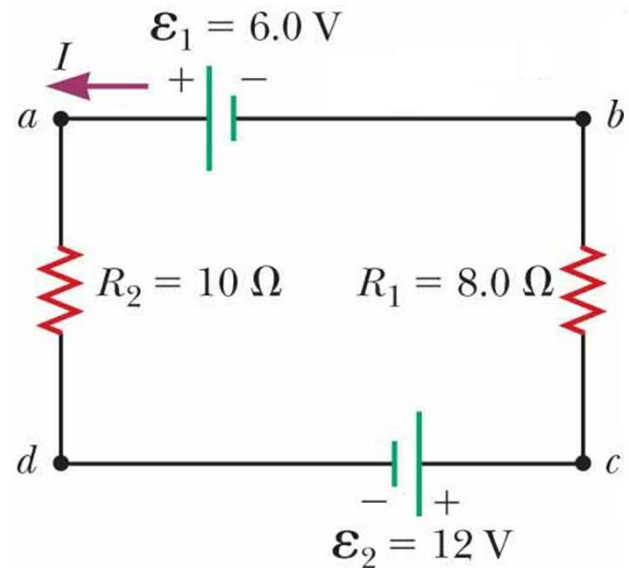
The negative sign for I indicates that the direction of the current is opposite the assumed direction.



Example 27.6: A Single-Loop Circuit

What if the polarity of the 12.0-V battery were reversed? How would that affect the circuit?

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1 + R_2} = \frac{6.0 \text{ V} + 12 \text{ V}}{8.0 \text{ } \Omega + 10 \text{ } \Omega} = \boxed{1.0 \text{ A}}$$



Example 27.6: A Single-Loop Circuit

Find the currents I_1 , I_2 , and I_3 in the circuit shown in the figure.

$$I_1 + I_2 - I_3 = 0$$

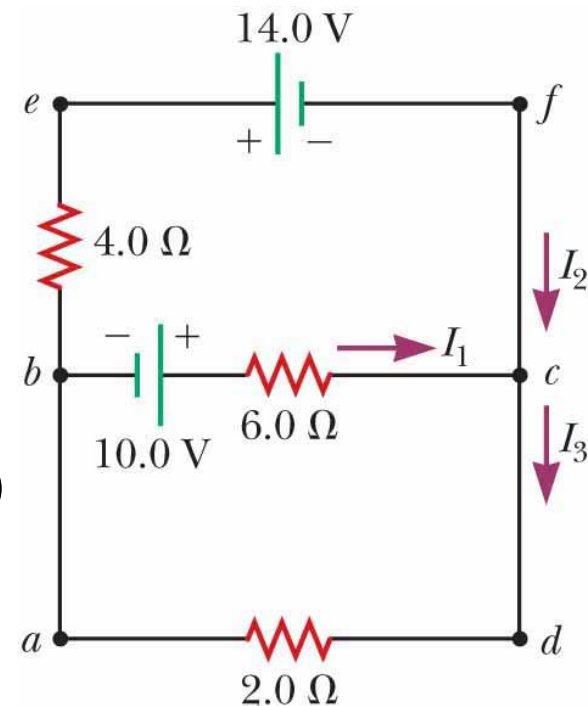
abcda:

$$10.0 \text{ V} - (6.0 \ \Omega)I_1 - (2.0 \ \Omega)I_3 = 0$$

befcb:

$$-(4.0 \ \Omega)I_2 - 14.0 \text{ V} + (6.0 \ \Omega)I_1 - 10.0 \text{ V} = 0$$

$$-24.0 \text{ V} + (6.0 \ \Omega)I_1 - (4.0 \ \Omega)I_2 = 0$$



Example 27.6: A Single-Loop Circuit

$$I_1 + I_2 - I_3 = 0 \quad \Rightarrow \quad I_3 = I_1 + I_2$$

$$10.0 \text{ V} - (6.0 \ \Omega)I_1 - (2.0 \ \Omega)(I_1 + I_2) = 0$$

$$10.0 \text{ V} - (8.0 \ \Omega)I_1 - (2.0 \ \Omega)I_2 = 0$$

$$4(-24.0 \text{ V} + (6.0 \ \Omega)I_1 - (4.0 \ \Omega)I_2 = 0)$$

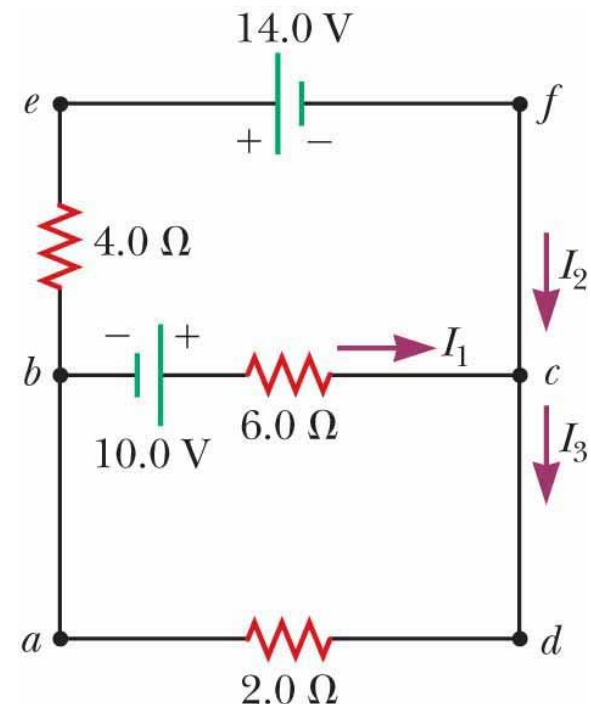
$$-96.0 \text{ V} + (24.0 \ \Omega)I_1 - (16.0 \ \Omega)I_2 = 0$$

$$3(10.0 \text{ V} + (8.0 \ \Omega)I_1 - (2.0 \ \Omega)I_2 = 0)$$

$$30.0 \text{ V} + (24.0 \ \Omega)I_1 - (6.0 \ \Omega)I_2 = 0$$

$$-66.0 \text{ V} - (22.0 \ \Omega)I_2 = 0$$

$$I_2 = \boxed{-3.0 \text{ A}}$$



Example 27.6: A Single-Loop Circuit

$$I_2 = -3.0 \text{ A} \Rightarrow$$

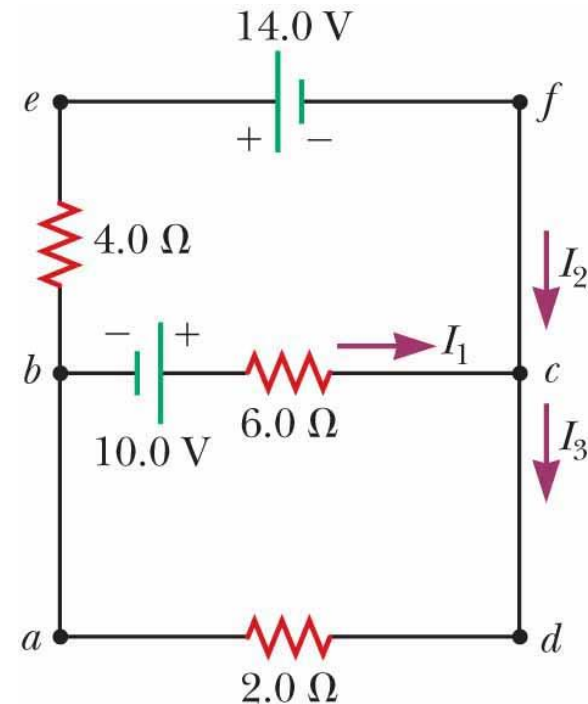
$$-24.0 \text{ V} + (6.0 \ \Omega) I_1 - (4.0 \ \Omega)(-3.0 \text{ A}) = 0$$

$$-24.0 \text{ V} + (6.0 \ \Omega) I_1 + 12.0 \text{ V} = 0$$

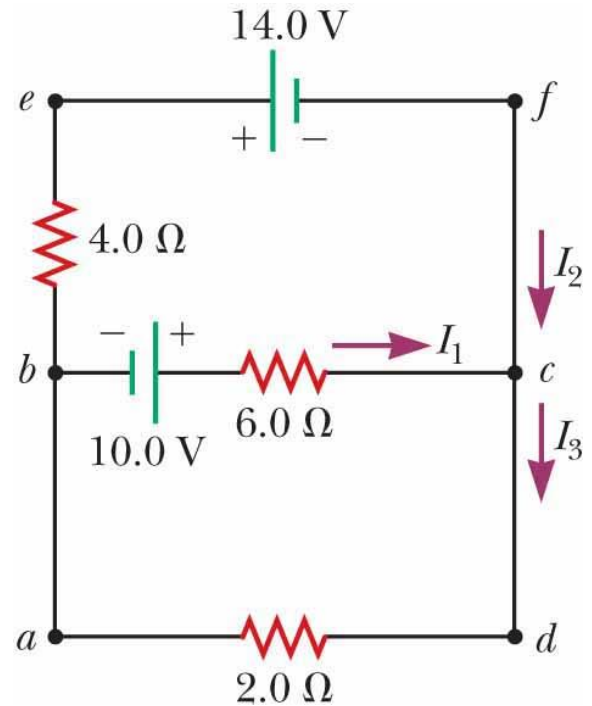
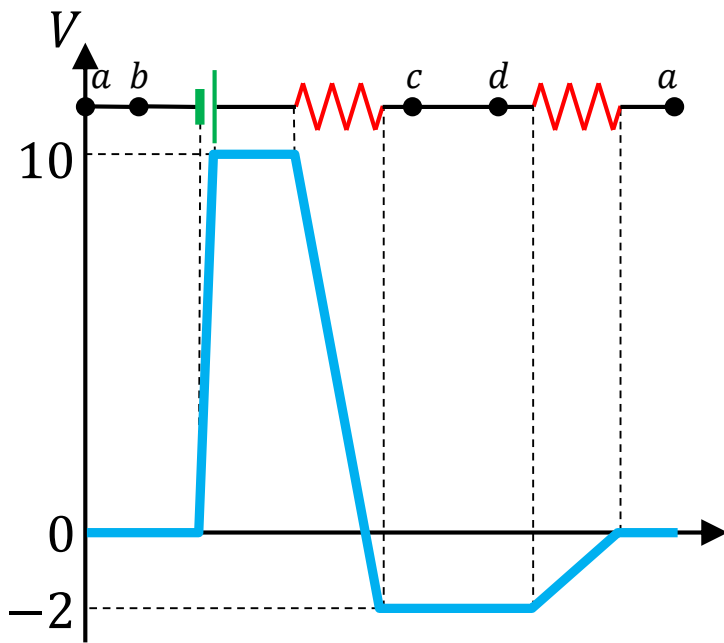
$$I_1 = \boxed{2.0 \text{ A}}$$

$$I_3 = I_1 + I_2 \Rightarrow$$

$$I_3 = 2.0 \text{ A} - 3.0 \text{ A} = \boxed{-1.0 \text{ A}}$$

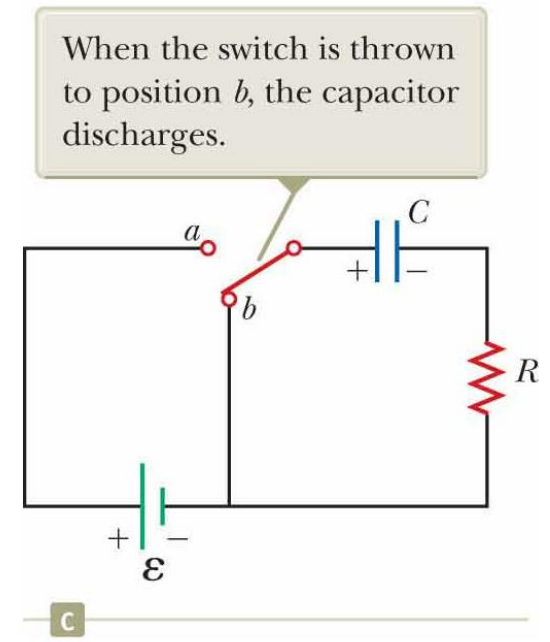
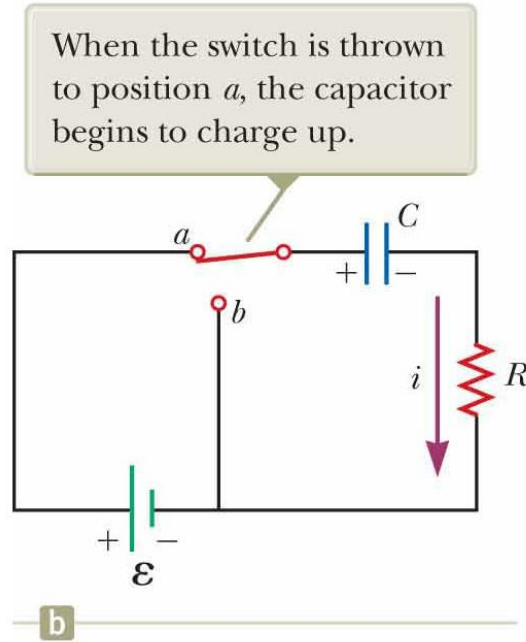
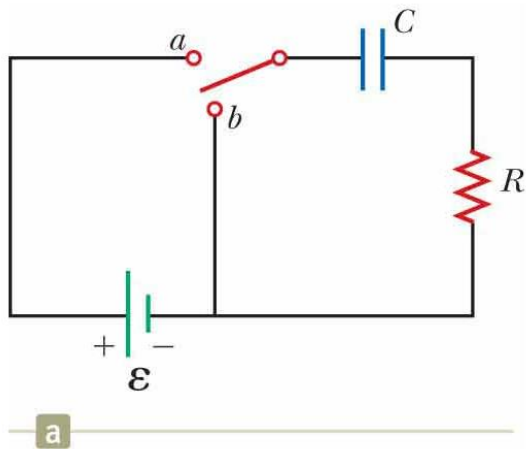


Example 27.6: A Single-Loop Circuit



Task: plot potential diagram for befcb path.

Charging a Capacitor



Charging a Capacitor

$$\varepsilon - \frac{q}{C} - iR = 0 \quad I_i = \frac{\varepsilon}{R} \quad (\text{current at } t = 0)$$

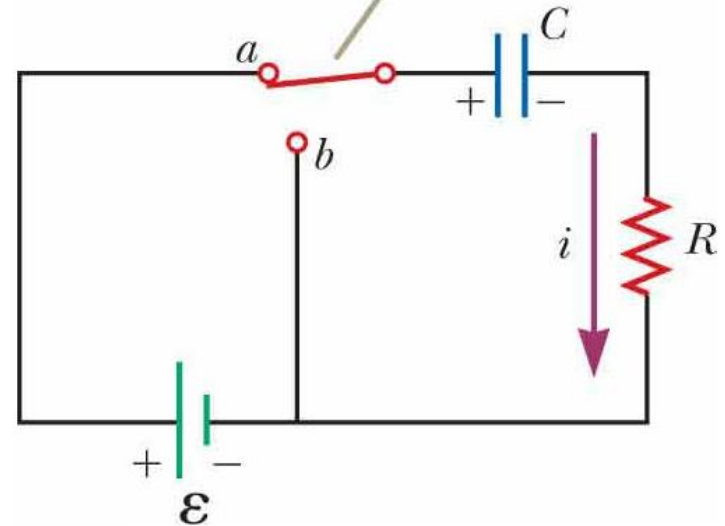
$$Q_{\max} = C\varepsilon \quad (\text{maximum charge})$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{1}{RC} dt$$

When the switch is thrown to position *a*, the capacitor begins to charge up.



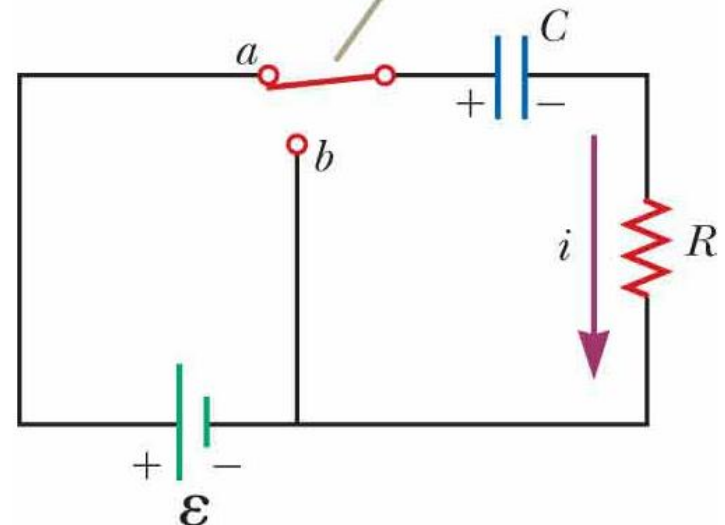
Charging a Capacitor

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC} \int_0^t dt \Rightarrow \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$q(t) = C\varepsilon\left(1 - e^{-t/RC}\right)$$
$$= Q_{\max}\left(1 - e^{-t/RC}\right)$$

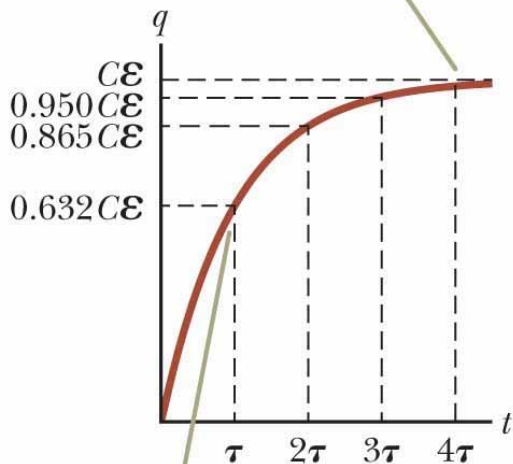
$$i(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

When the switch is thrown to position *a*, the capacitor begins to charge up.



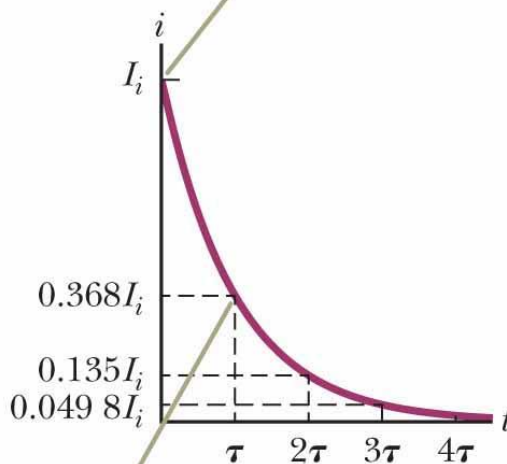
Charging a Capacitor

The charge approaches its maximum value $C\mathcal{E}$ as t approaches infinity.



After a time interval equal to one time constant τ has passed, the charge is 63.2% of the maximum value $C\mathcal{E}$.

The current has its maximum value $I_i = \mathcal{E}/R$ at $t = 0$ and decays to zero exponentially as t approaches infinity.



After a time interval equal to one time constant τ has passed, the current is 36.8% of its initial value.

$$\tau = RC$$

$$i = e^{-1} I_i = 0.368 I_i$$

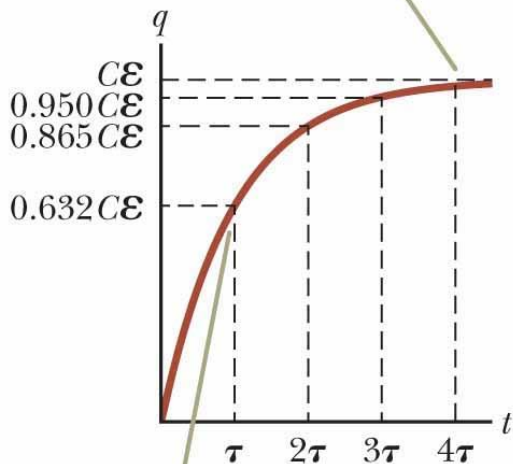
$$i = e^{-2} I_i = 0.135 I_i$$

a

b

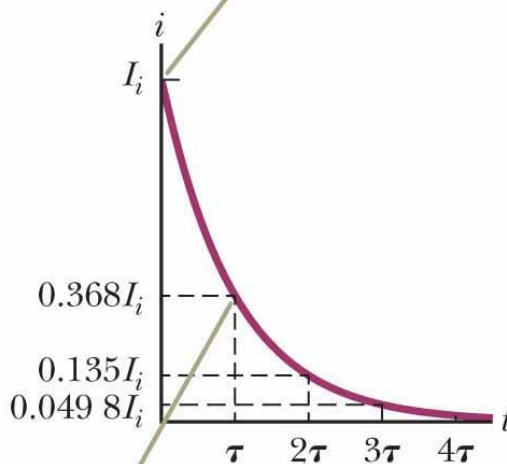
Charging a Capacitor

The charge approaches its maximum value $C\mathcal{E}$ as t approaches infinity.



After a time interval equal to one time constant τ has passed, the charge is 63.2% of the maximum value $C\mathcal{E}$.

The current has its maximum value $I_i = \mathcal{E}/R$ at $t = 0$ and decays to zero exponentially as t approaches infinity.



After a time interval equal to one time constant τ has passed, the current is 36.8% of its initial value.

$$C\mathcal{E} \left[1 - e^{-1} \right] = 0.632C\mathcal{E}$$

$$Q_{\max} \mathcal{E} = C\mathcal{E}^2$$

$$\frac{1}{2} Q_{\max} \mathcal{E} = \frac{1}{2} C\mathcal{E}^2$$

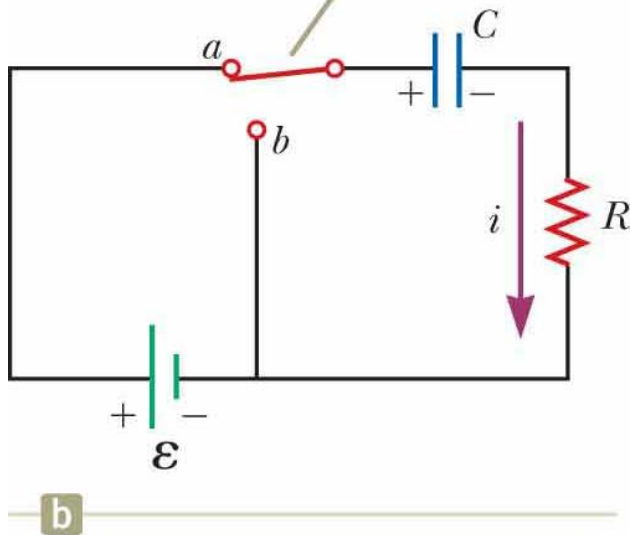
a

b

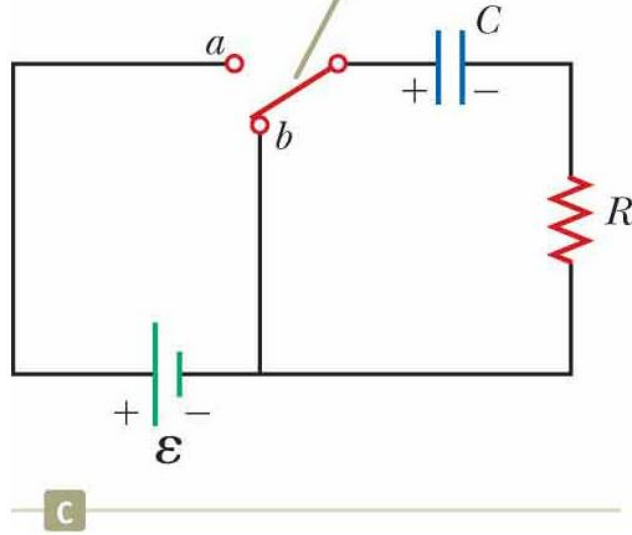
Discharging a Capacitor

$$-\frac{q}{C} - iR = 0 \quad -R \frac{dq}{dt} = \frac{q}{C} \Rightarrow \frac{dq}{q} = -\frac{1}{RC} dt$$

When the switch is thrown to position *a*, the capacitor begins to charge up.



When the switch is thrown to position *b*, the capacitor discharges.



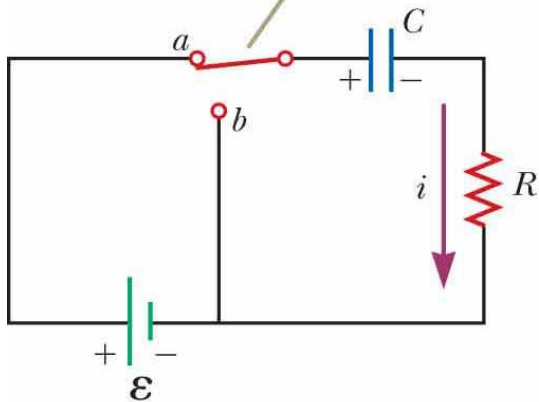
Discharging a Capacitor

$$\int_{Q_i}^q \frac{dq}{q} = -\frac{1}{RC} \int_0^t dt \Rightarrow \ln\left(\frac{q}{Q_i}\right) = -\frac{t}{RC}$$

$$q(t) = Q_i e^{-t/RC}$$

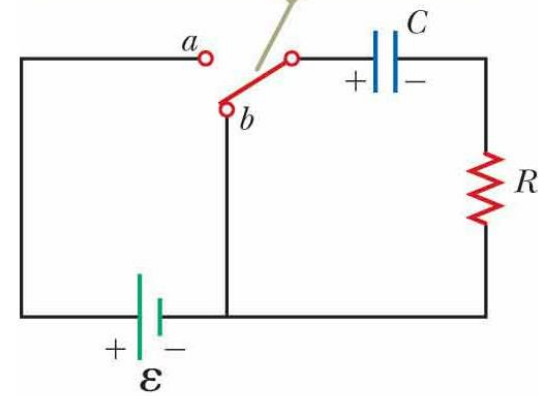
$$i(t) = -\frac{Q_i}{RC} e^{-t/RC}$$

When the switch is thrown to position *a*, the capacitor begins to charge up.



b

When the switch is thrown to position *b*, the capacitor discharges.



c