Current and Resistance

Physics for Scientists and Engineers, 10e
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Electric Current

- Flow of charges between two points in space is driven by potential difference between points
- Amount of current depends on:
  - Potential difference
  - Properties of any material that may fill space through which charges flow

- Analogy between water flow and current
  - Flow of water in plumbing pipe driven by pressure difference
  - Can be quantified by specifying amount of water that emerges from faucet during given time interval, measured in liters per minute
  - River current can be characterized by describing rate at which water flows past particular location

- Example: flow over the brink at Niagara Falls

  $\rightarrow$ rates between 1 400 m^3/s and 2 800 m^3/s

Electric Current

Charges are moving perpendicular to surface of area *A*

Current is defined as rate at which charge flows through this surface

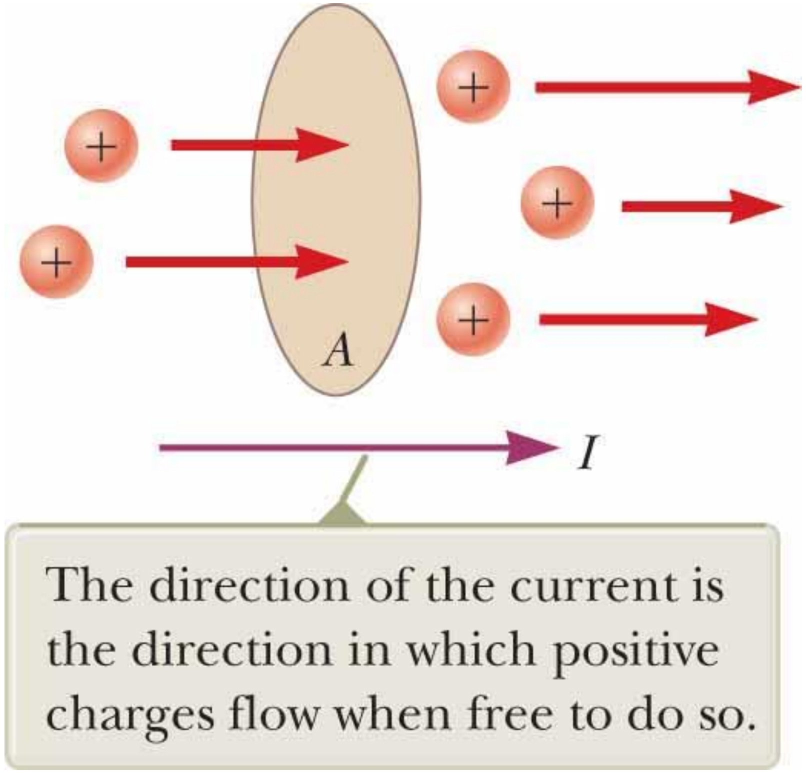
Average current I_{avg}:

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t}$$

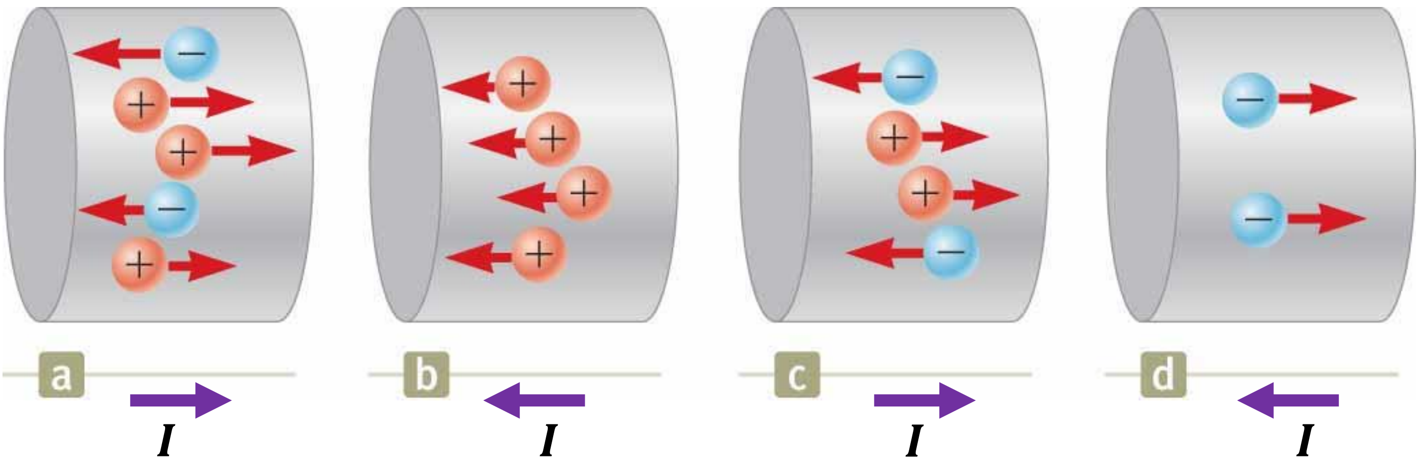
Instantaneous current *I*:

$$I \equiv \frac{dQ}{dt}$$

$$1\ \text{A} = 1\ \text{C/s}$$



Electric Current



$I_a > I_b = I_c > I_d$

- Charged particles passing through surface in figure can be positive, negative, or both

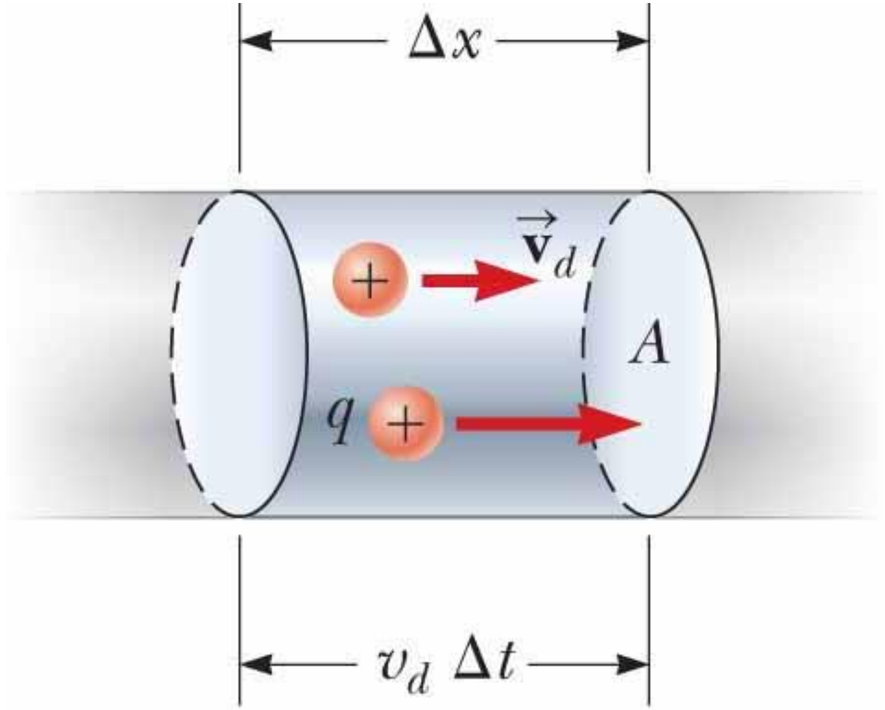
- Conventional to assign to current a direction that is same as that of flow of positive charge
    - In electrical conductors (e.g., copper or aluminum) current results from motion of negatively charged electrons: in an ordinary conductor direction of current is opposite direction of flow of electrons
    - For beam of positively charged protons in accelerator → current in direction of motion of protons
    - In some cases (i.e., involving gases and electrolytes): current result of flow of both positive and negative charges

Microscopic Model of Current

$$\Delta Q = Nq = (nA\Delta x)q$$

$$= (nAv_d \, \Delta t)q$$

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = nqv_d A$$



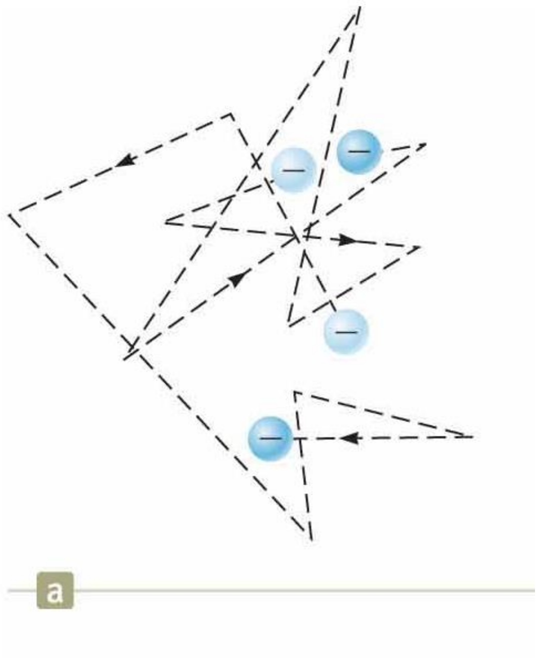
n – charge carrier density (number of mobile charge carriers per unit volume)
q – charge on each carrier
v_d –carriers velocity, parallel to axis of cylinder
A – cross-section area of cylindrical conductor

Microscopic Model of Current

Speed of charge carriers v_d = average speed: **drift speed**

Consider conductor in which charge carriers are free electrons :

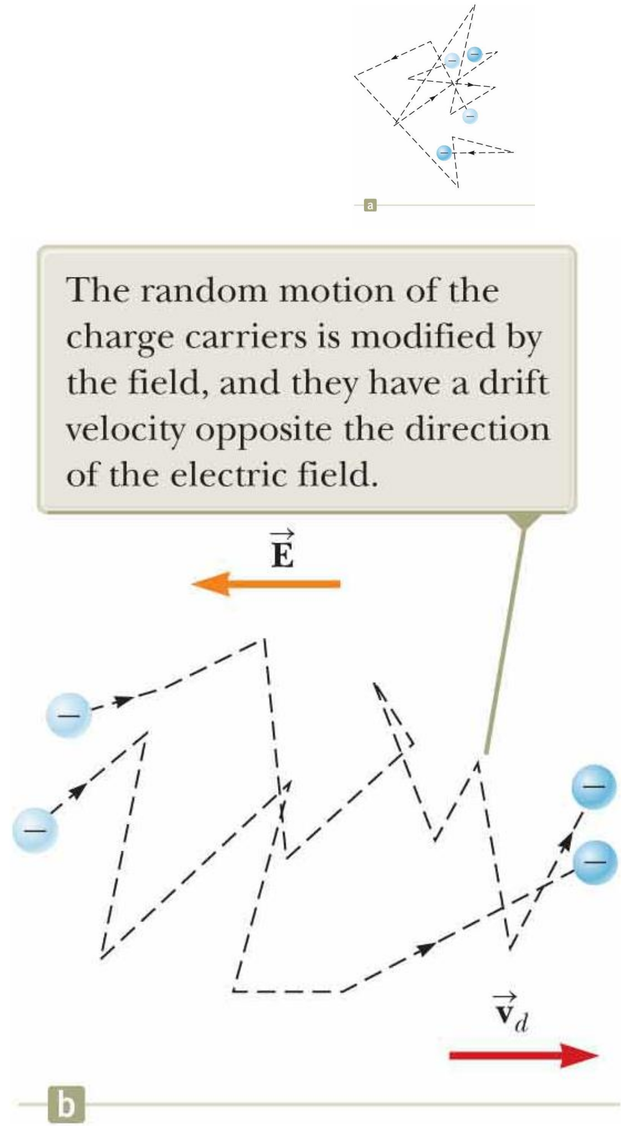
- These electrons undergo random thermal motion analogous to motion of gas molecules, with $v_{th} \approx 10^6$ m/s
- Electrons collide repeatedly with metal atoms
- Resultant motion complicated and zigzagged
- Drift speed $v_d = 0$



Microscopic Model of Current

- When potential difference applied across conductor (e.g. by means of a battery):
  - Electric field set up in conductor
  - Field exerts electric force $\vec{F} = q\vec{E} = -e\vec{E}$ on electrons, producing a current
- In addition to zigzag motion due to collisions with metal atoms ($v_{th} \approx 10^6$ m/s):
  - Electrons move slowly along conductor (direction opposite **E**) at **drift velocity** $v_d \approx 10^{-6} \div 10^{-4}$ m/s

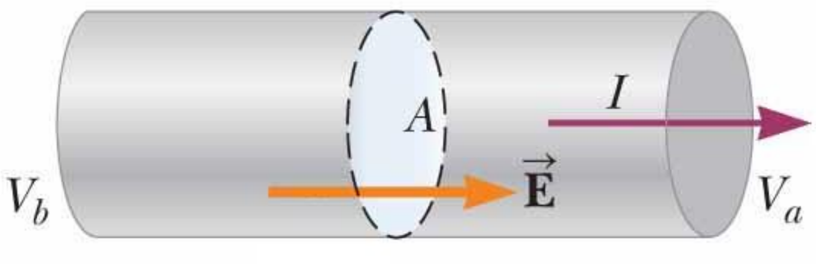
- Think of atom–electron collisions in conductor as effective internal friction (or drag force)
- Energy transferred from electrons to metal atoms during collisions causes increase in atom's vibrational energy → Corresponding increase in conductor's temperature



Resistance

Current:

$$I = nqv_d A$$



Current density:

$$J \equiv \frac{I}{A} = nqv_d$$

For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current:

$$J = \sigma E$$

σ – conductivity of conductor

Resistance

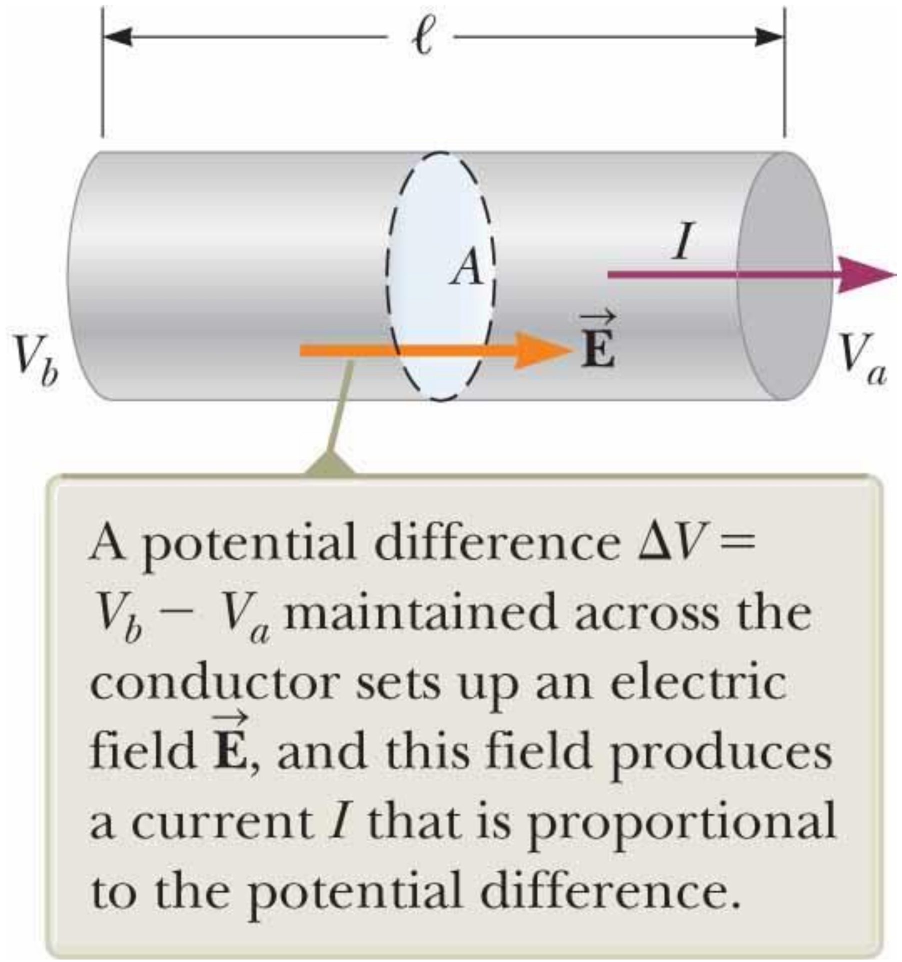
$$\Delta V = E\ell$$

$$\Delta V = \frac{J}{\sigma}\ell$$

$$\Delta V = \left(\frac{\frac{I}{A}}{\sigma}\right)\ell = \left(\frac{\ell}{\sigma A}\right)I = RI$$

$$R \equiv \frac{\Delta V}{I}$$

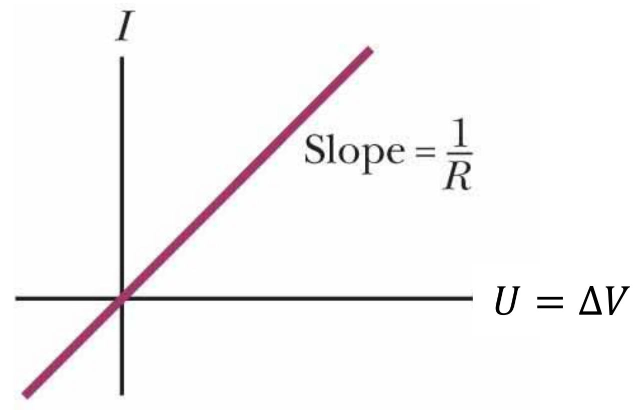
$$1\ \Omega \equiv 1\ \mathrm{V/A}$$



Ohm's law

1. The current through a conductor between two points is directly proportional to the voltage across the two points with the resistance as the constant of proportionality:

$$I = \frac{1}{R} U$$



2. For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current:

$$J = \sigma E$$

Resistors

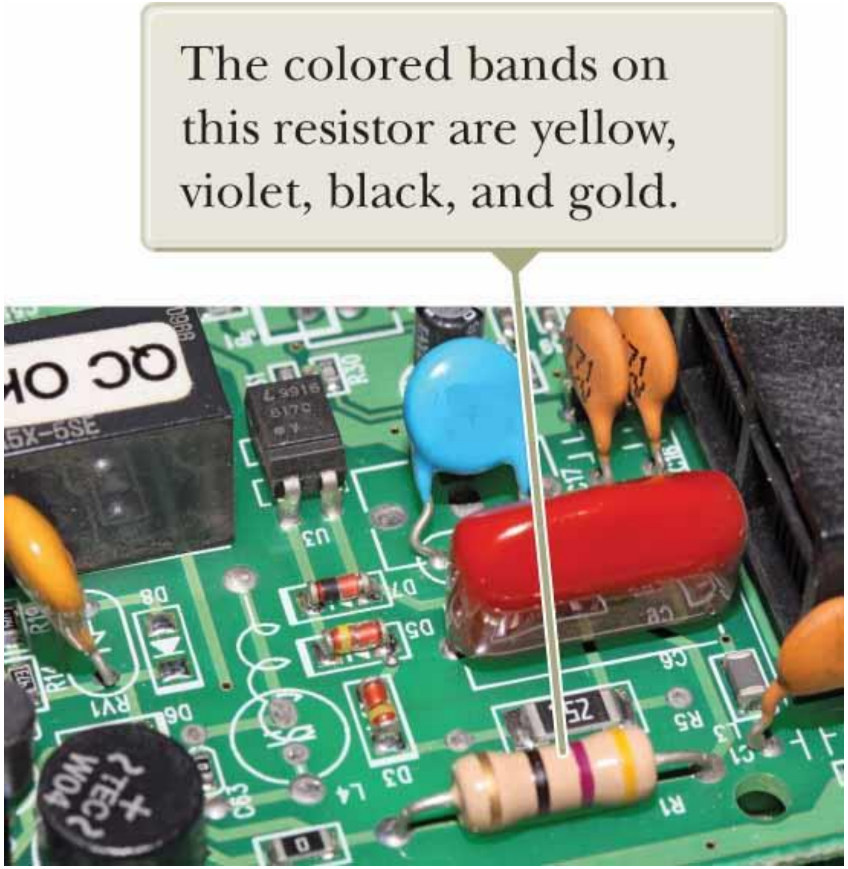


TABLE 26.1 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

$$R = \left(10C_1 + C_2\right) \times 10^{C_3}\,\Omega$$

$$\Delta R = R \cdot C_4$$

Resistance value: $R_{YVBG} = 47 \times 10^0\ \Omega = 47\ \Omega$

Tolerance value: $\Delta R_{YVBG} = R \cdot 5\% = 2\ \Omega$

Resistivity

Inverse of conductivity is **resistivity** ρ:

$$\rho = \frac{1}{\sigma}$$

$$R = \frac{\ell}{\sigma A}$$

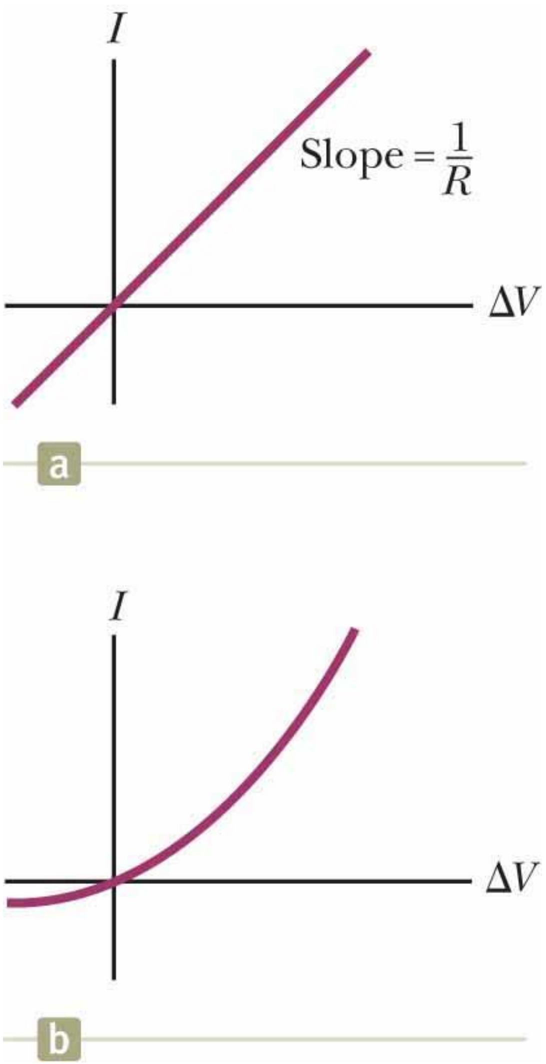
$$R = \rho \frac{\ell}{A}$$

TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [$(^\circ C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and $1.50 \times 10^{-6}\ \Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Ohmic and Nonohmic Materials



Nonohmic materials have nonlinear current–potential difference relationship

- diode
- transistor
- thermistor
- filament lamp
- …

Drude Model for Electrical Conduction

1) In absence of electric field: conduction electrons move in random directions through conductor

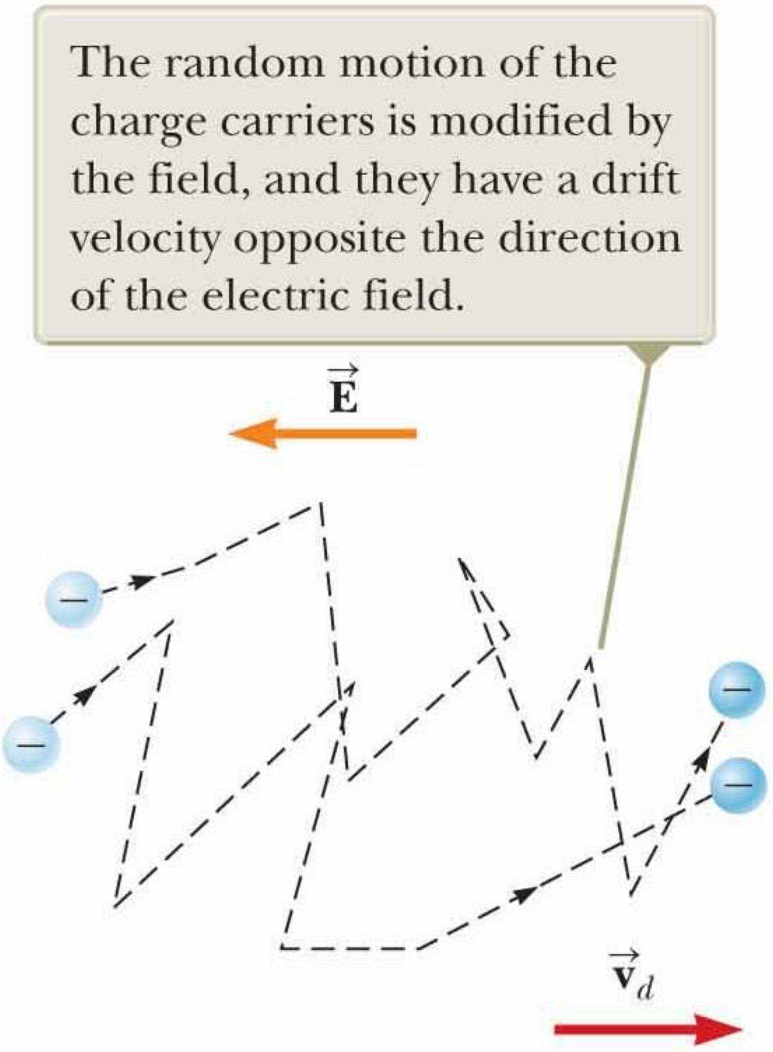
- Situation similar to motion of gas molecules confined in a vessel
- Conduction electrons in a metal sometimes referred to as *electron gas*

2) When electric field applied to system → free electrons drift slowly in direction opposite that of electric field (figure)

- Average drift speed v_d much smaller (typically 10^{-4} m/s) than average speed v_{avg} between collisions (typically 10^6 m/s)

3) Electron's motion after collision independent of its motion before collision

- Excess energy acquired by electrons due to work done on them by electric field transferred to atoms of conductor when electrons and atoms collide
- Energy transferred to atoms causes internal energy of system and temperature of conductor to increase



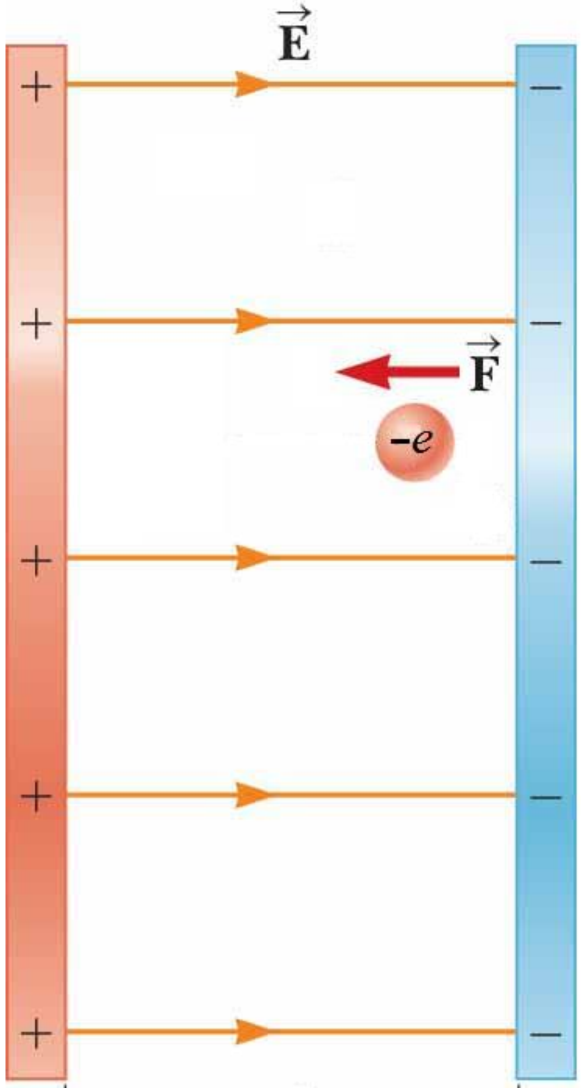
Drude Model for Electrical Conduction

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{q\vec{\mathbf{E}}}{m_e}$$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \vec{\mathbf{v}}_i + \frac{q\vec{\mathbf{E}}}{m_e}t$$

$$\vec{\mathbf{v}}_{f,\text{avg}} = \vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e}\tau$$

$$I_{\text{avg}} = nq\left(\frac{qE}{m_e}\tau\right)A = \frac{nq^2\tau A}{m_e}E$$



Drude Model for Electrical Conduction

$$J = \frac{nq^2\tau}{m_e} E \qquad\qquad J = \sigma E$$

$$\sigma = \frac{nq^2\tau}{m_e}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

$$\tau = \frac{\ell_{\text{avg}}}{v_{\text{avg}}}$$

Resistance and Temperature

$$\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$$

$$\alpha = \frac{\Delta\rho/\rho_0}{\Delta T}$$

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right]$$

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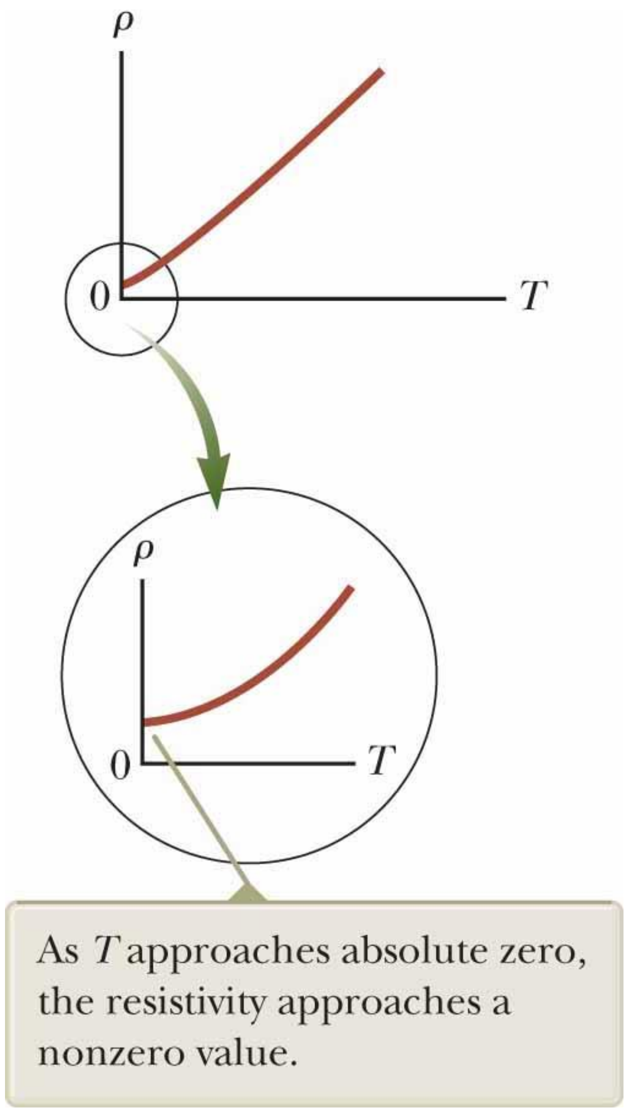
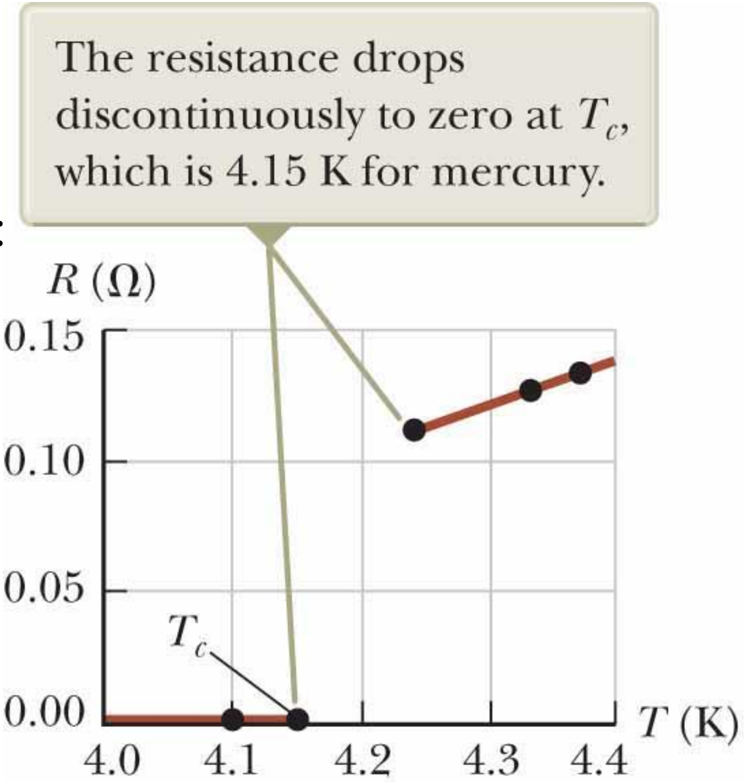
Resistance and Temperature

- For some metals (i.e., copper): resistivity nearly proportional to temperature (figure)
  - Nonlinear region always exists at very low temperatures
  - Resistivity usually reaches some finite value as temperature approaches absolute zero
- Residual resistivity near absolute zero caused primarily by collision of electrons with impurities and imperfections in metal
- High-temperature resistivity (linear region) predominantly characterized by collisions between electrons and metal atoms



Superconductors

- **Superconductors:** class of metals and compounds whose resistance decreases to zero below certain temperature T_c (**critical temperature**)
- Resistance–temperature graph for superconductor:
  - Normal metal at temperatures above T_c
- When temperature at or below T_c:
  - Resistivity drops suddenly to zero
- Discovered in 1911 by Dutch physicist Heike Kamerlingh-Onnes
  - Working with mercury → superconductor below 4.2 K
- Resistivities of superconductors below their T_c values $< 4 \times 10^{-25}\ \Omega \cdot m$
  - $\approx 10^{17}$ times smaller than resistivity of copper
  - In practice: resistivities are considered to be zero



Superconductors

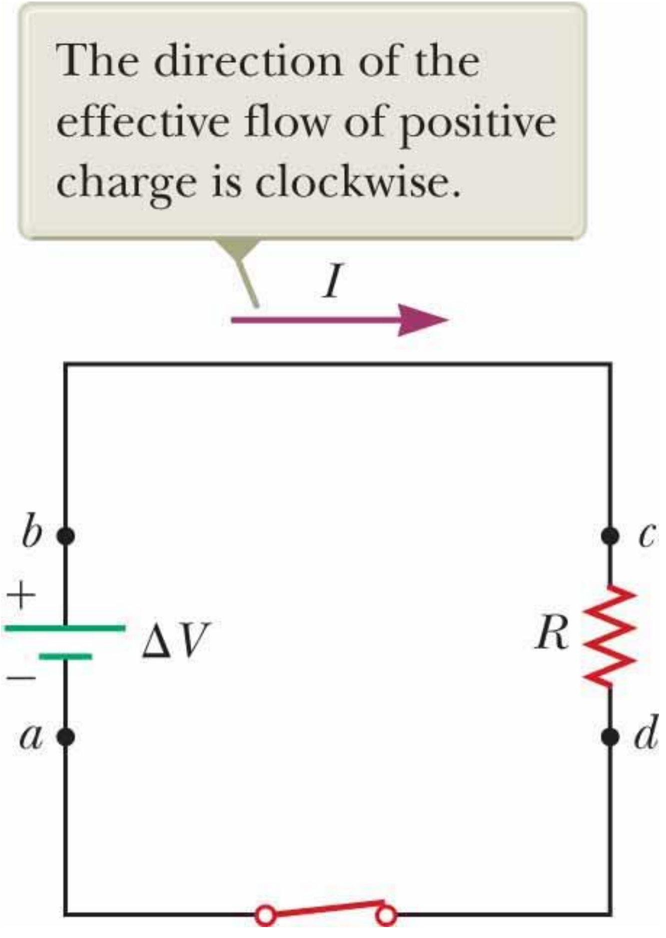
TABLE 26.3 Critical Temperatures for Various Superconductors

Material	T_c (K)
$HgBa_2Ca_2Cu_3O_8$	134
Tl—Ba—Ca—Cu—O	125
Bi—Sr—Ca—Cu—O	105
$YBa_2Cu_3O_7$	92
Nb_3Ge	23.2
Nb_3Sn	18.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88

Power in electric circuits

As a result of current I, the amount of charge $\mathrm{d}q$ moves between c and d in time $\mathrm{d}t$, through a decrease in potential of magnitude ΔV, and thus its electric potential energy decreases in magnitude by amount:

$$\mathrm{d}U_E = \mathrm{d}q\Delta V = I\mathrm{d}t\Delta V$$



The principle of conservation of energy tell as that the decrease in electric potential energy is accompanied by a transfer of energy to some other form.
The power P associated with that transfer is the rate of transfer:

$$P = \frac{\mathrm{d}U_E}{\mathrm{d}t} = I\Delta V = IU$$

Power in electric circuits

$$P = IU$$

This power P is the rate of energy transfer from the battery to the device, e.g.:

- motor: mechanical work
- charger: stored chemical energy of rechargeable battery
- resistor: internal thermal energy:

$$P_R = IU = \frac{U^2}{R} = I^2 R$$

Joule–Lenz law: the power of heating generated by an electrical conductor is proportional to the product of its resistance and the square of the current:

Electrical Power

Why is energy transported through electrical wires at very high voltages?

Electrical Power

- Energy is transported by electricity through power lines with non zero resistance

- Utility companies seek to minimize energy transformed to internal energy in lines and maximize energy delivered to consumer

- Same amount of useful energy $P = IU$ can be transported either at:
    - High currents and low potential differences
    - Low currents and high potential differences

- Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons:
    - Copper wire very expensive $\rightarrow$ cheaper to use high-resistance wire (small cross-sectional area)
    - $P_R = I^2 R$ loss is reduced by keeping current I as low as possible
    - Transferring energy at high voltage