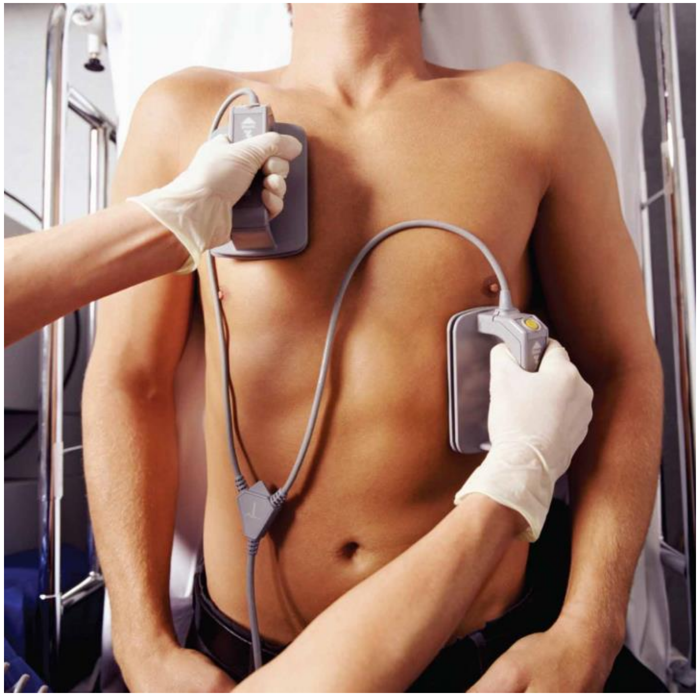
Storyline

Chapter 25:
Capacitance and Dielectrics

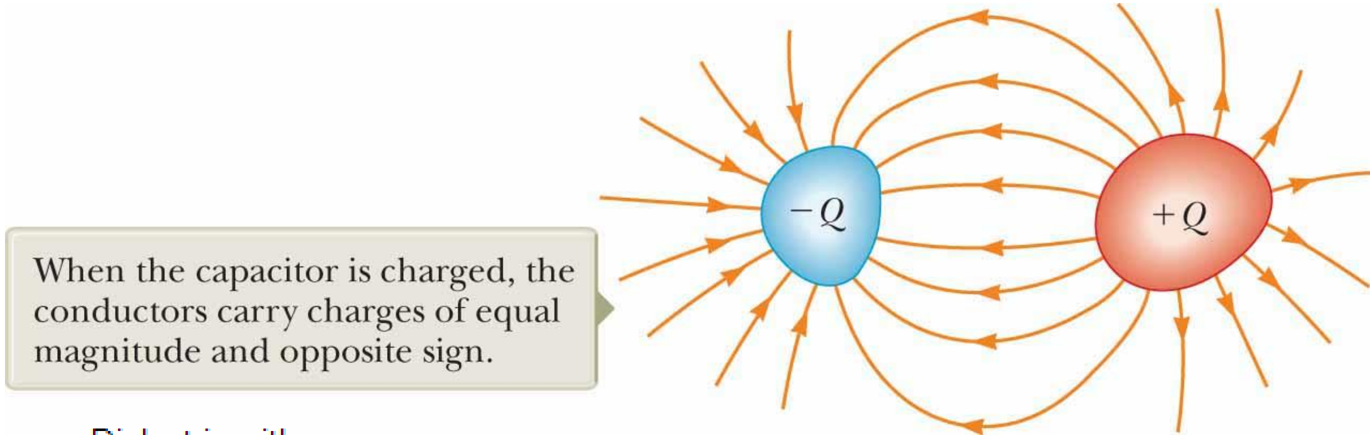


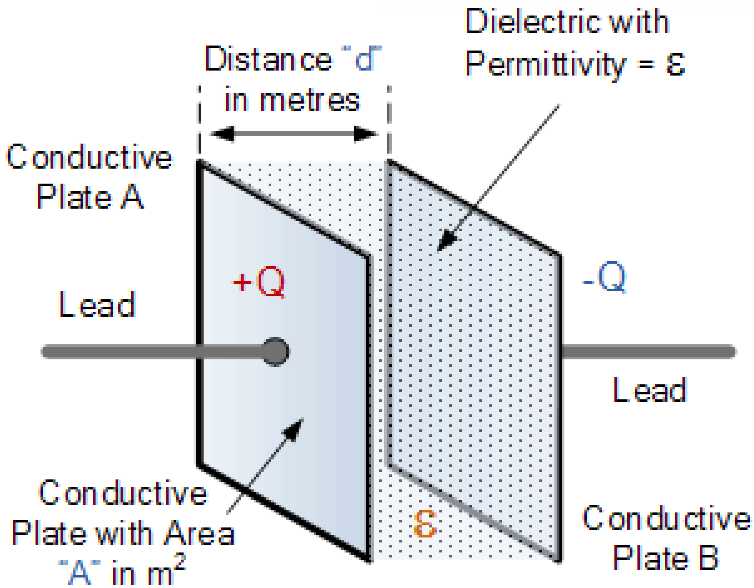
Physics for Scientists and Engineers, 10e
Raymond A. Serway
John W. Jewett, Jr.

BROOKS/COLE
CENGAGE Learning™

Capacitor

Capacitor: combination of two conductors, called plates





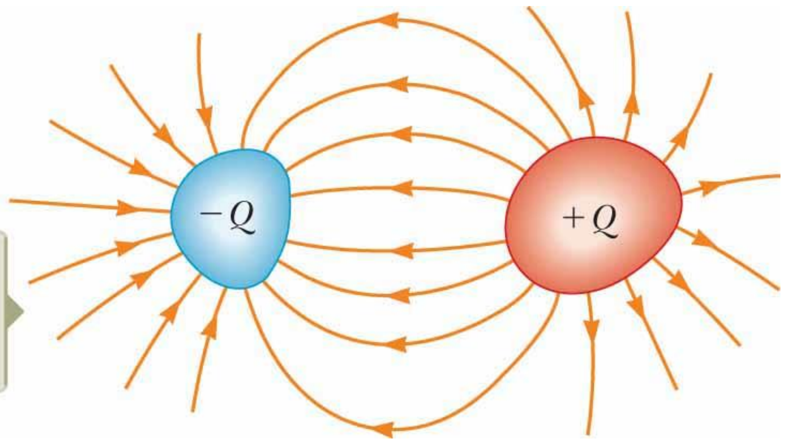
Definition of Capacitance

$$Q = C\Delta V$$

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$

$$1\ \text{F} = 1\ \text{C/V}$$



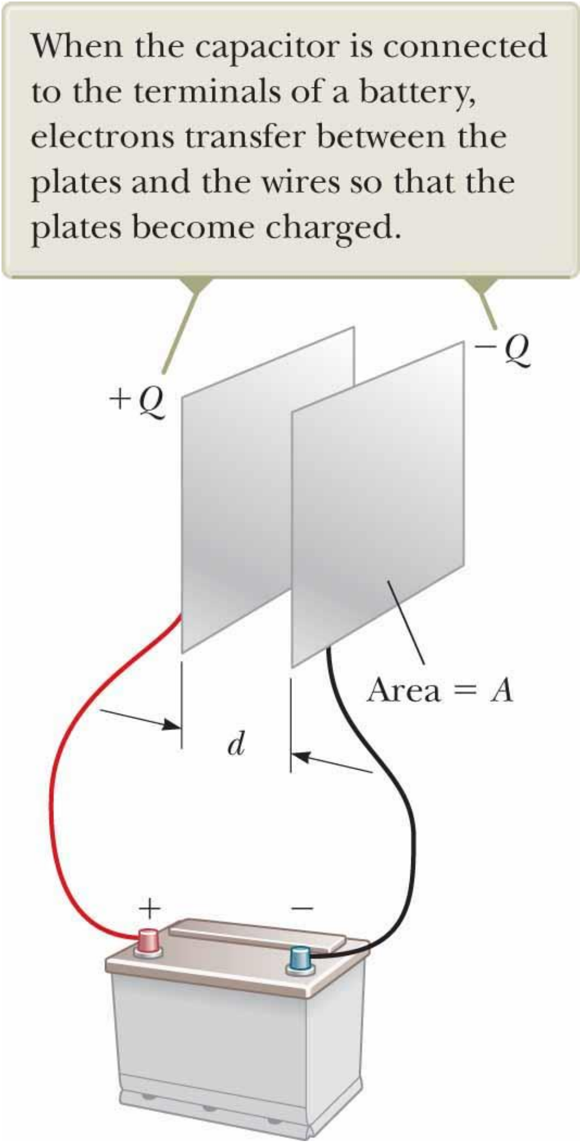
When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.

Definition of Capacitance

Consider capacitor formed from pair of parallel plates:
- Each plate connected to one terminal of battery
  - Acts as source of potential difference
- If capacitor initially uncharged:
  - Battery establishes electric field in connecting wires

For plate connected to negative terminal of battery:
- Electric field in wire applies force on electrons in wire immediately outside this plate:
  - Force causes electrons to move onto plate
  - Movement continues until plate, wire, and terminal all at same electric potential
- Once equilibrium attained potential difference no longer exists between terminal and plate:
  - Result: no electric field present in wire and electrons stop moving
- Plate now carries negative charge

- Similar process occurs at other capacitor plate:
  - Electrons move from plate to wire, leaving plate positively charged



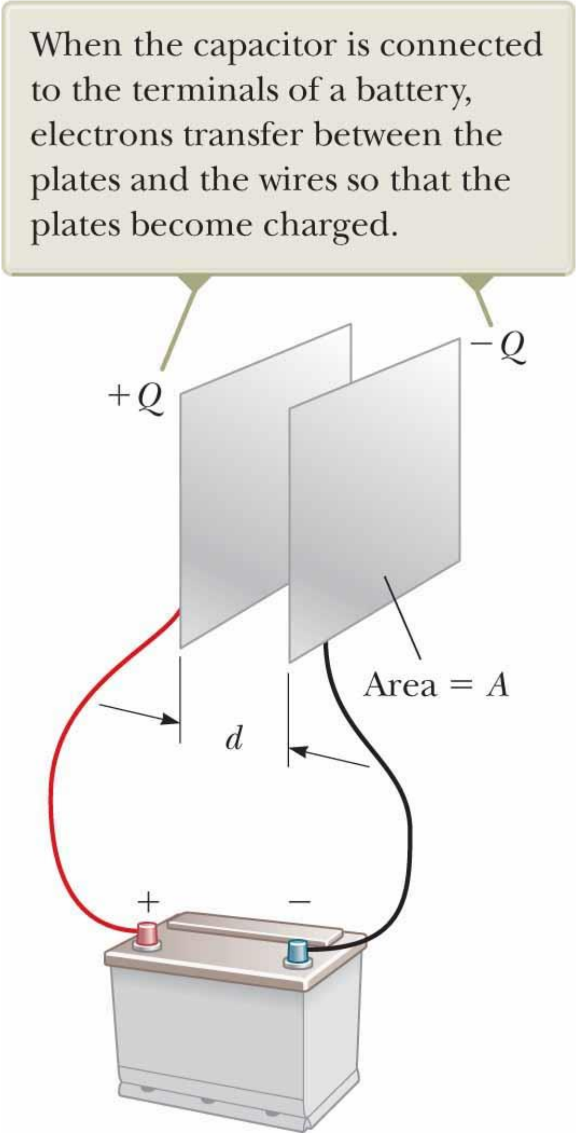
Calculating Capacitance

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\varepsilon_0 A}$$

$$C = \frac{\varepsilon_0 A}{d}$$

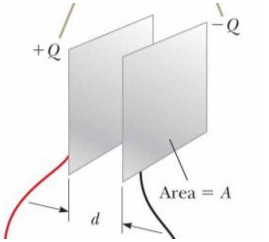


Capacitance

The **parallel-plate** capacitor:

- Proportional to area of plates
- Inversely proportional to plate separation

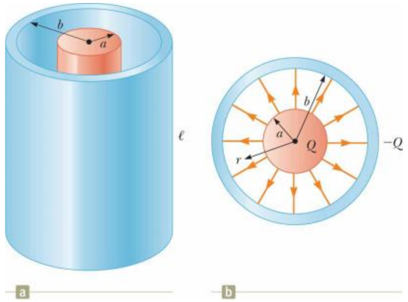
$$C = \frac{\varepsilon_0 A}{d}$$



The **cylindrical** capacitor:

- depends on the radii *a* and *b*
- is proportional to the length of the cylinders

$$C = \frac{4\pi\varepsilon_0 L}{2\ln\left(b/a\right)}$$



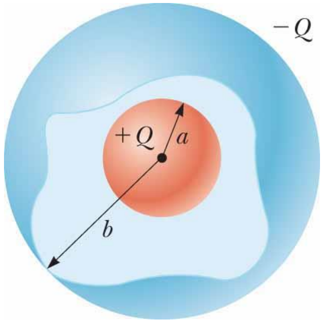
An example of this type of geometric arrangement is a coaxial cable, which consists of two concentric cylindrical conductors separated by an insulator. You probably have a coaxial cable attached to your television set if you are a subscriber to cable television. The coaxial cable is especially useful for shielding electrical signals from any possible external influences.

The **spherical** capacitor:

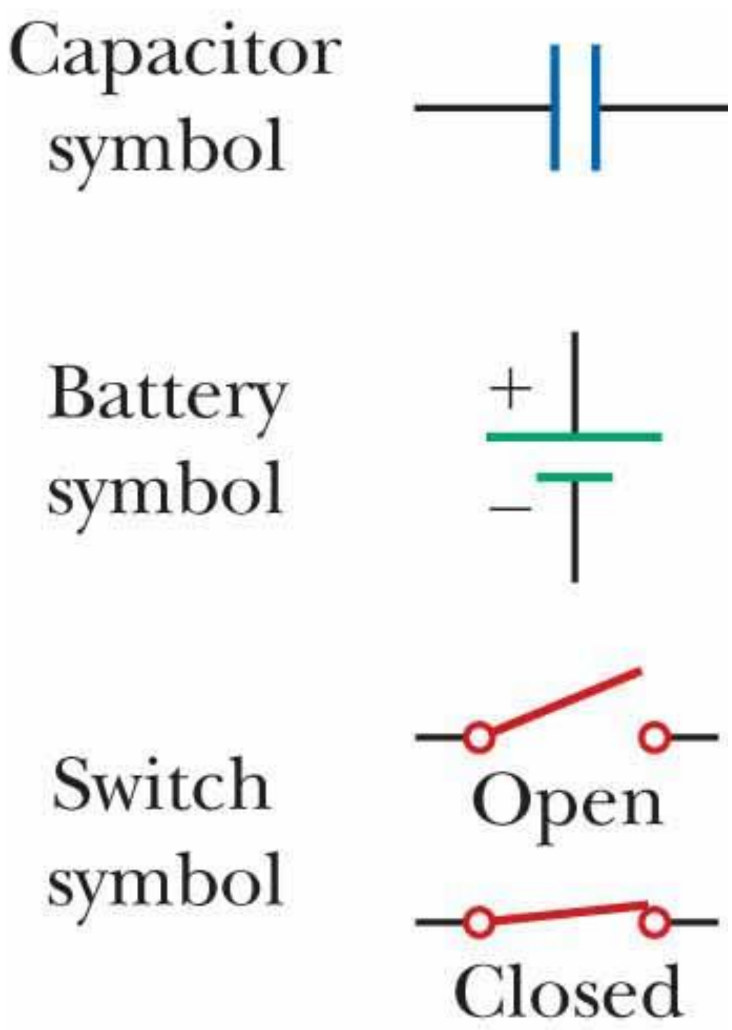
- depends on the radii *a* and *b*

$$C = \frac{4\pi\varepsilon_0 ab}{\left(b-a\right)}$$

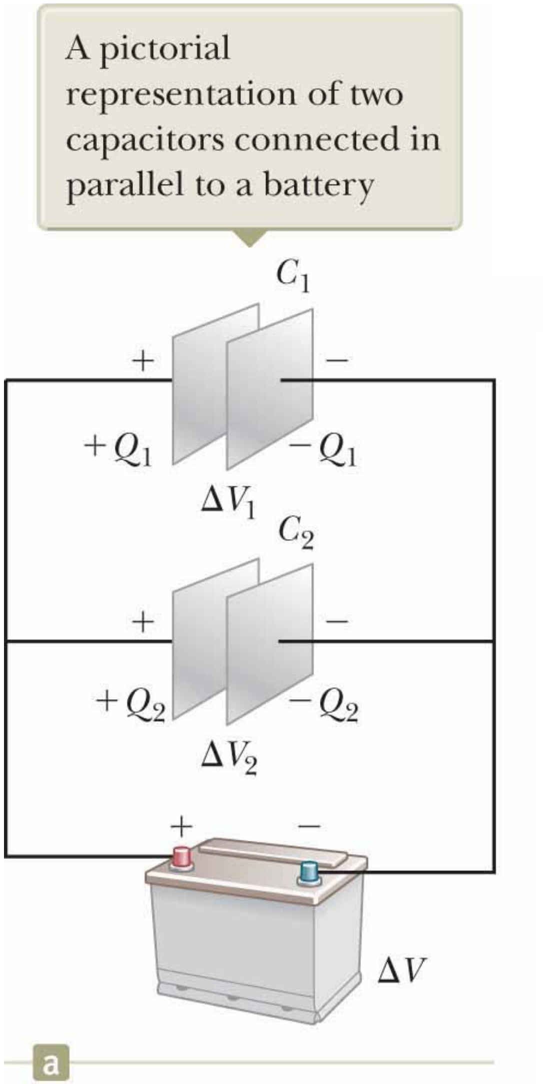
$$C_{b\to\infty} = 4\pi\varepsilon_0 a$$



Combinations of Capacitors



Parallel Combination

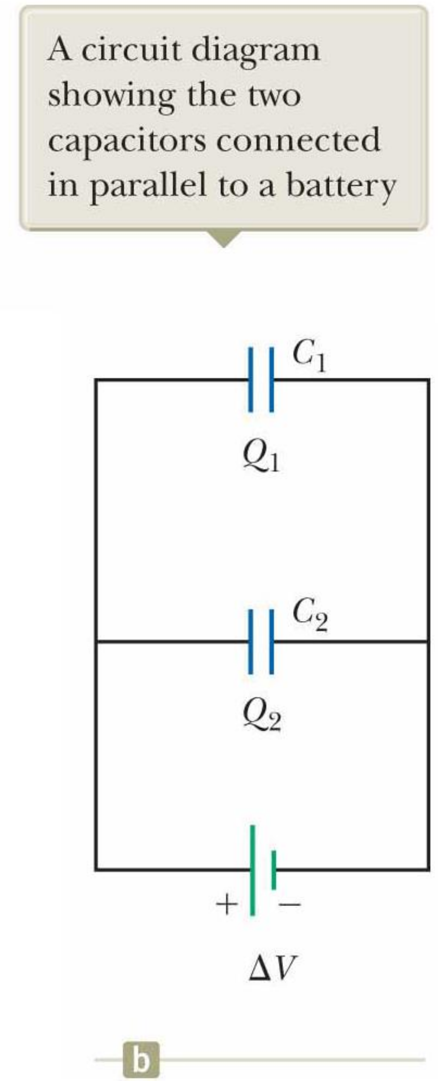


$$\Delta V_1 = \Delta V_2 = \Delta V$$

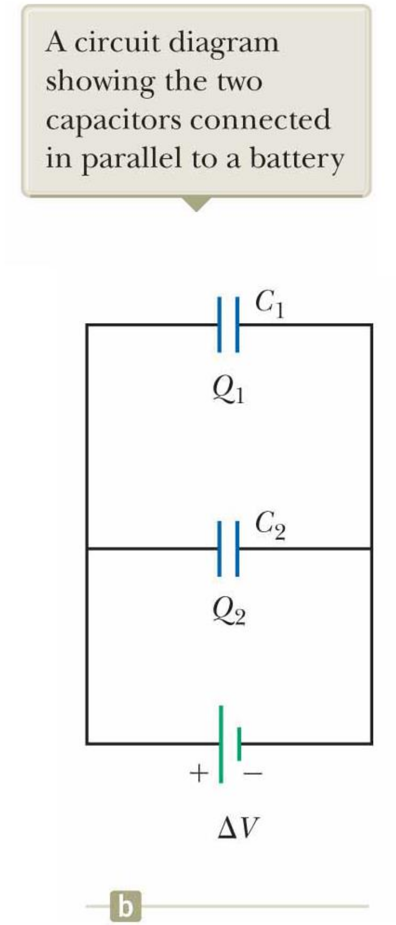
$$Q_{tot} = Q_1 + Q_2$$

$$\left(C = \frac{Q}{\Delta V} \right)$$

$$= C_1 \Delta V_1 + C_2 \Delta V_2$$



Parallel Combination

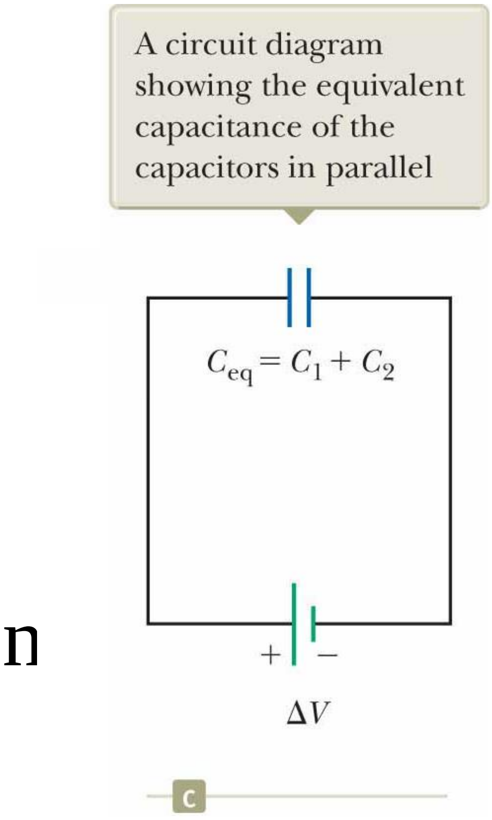


$$Q_{\text{tot}} = C_1 \Delta V_1 + C_2 \Delta V_2$$

$$Q_{\text{tot}} = C_{\text{eq}} \Delta V$$

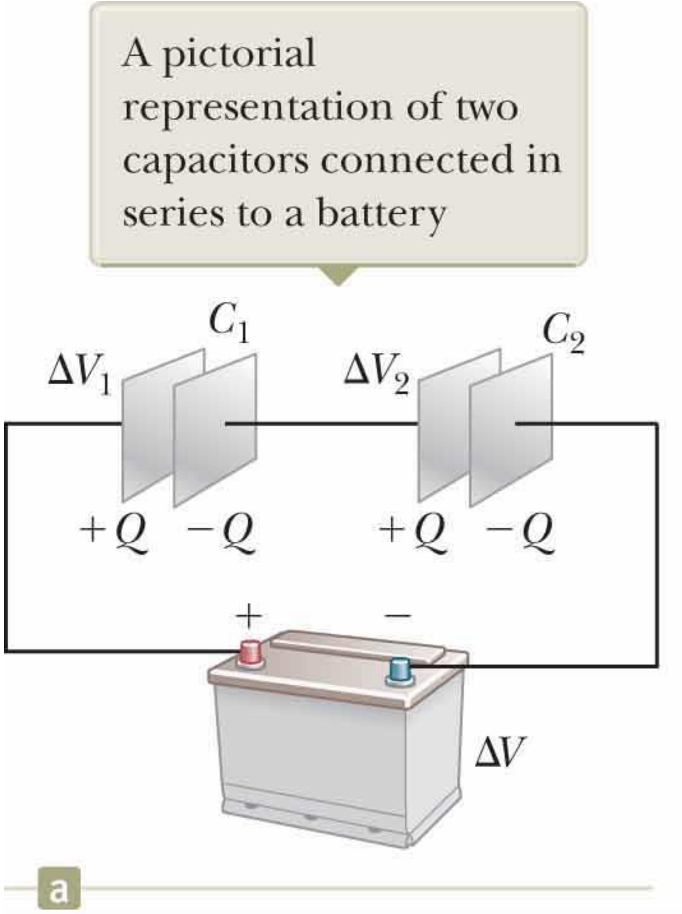
$$C_{\text{eq}} = C_1 + C_2$$

$$\left(\text{parallel combination}\right.$$



$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots \quad (\text{parallel combination})$$

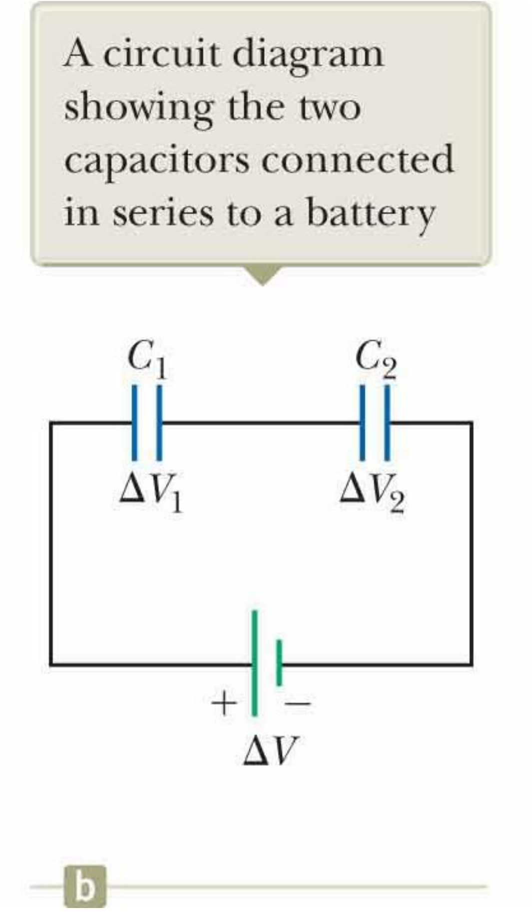
Series Combination



$$Q_1 = Q_2 = Q$$

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2$$

$$= \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$



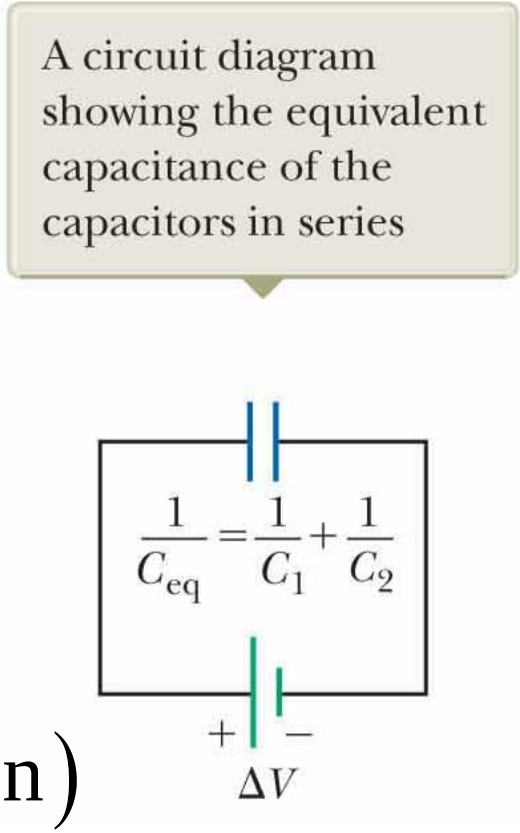
Series Combination

$$\Delta V_{\text{tot}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\Delta V_{\text{tot}} = \frac{Q}{C_{\text{eq}}}$$

$$\frac{Q}{C_{\text{eq}}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series combination)}$$



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \text{(series combination)}$$

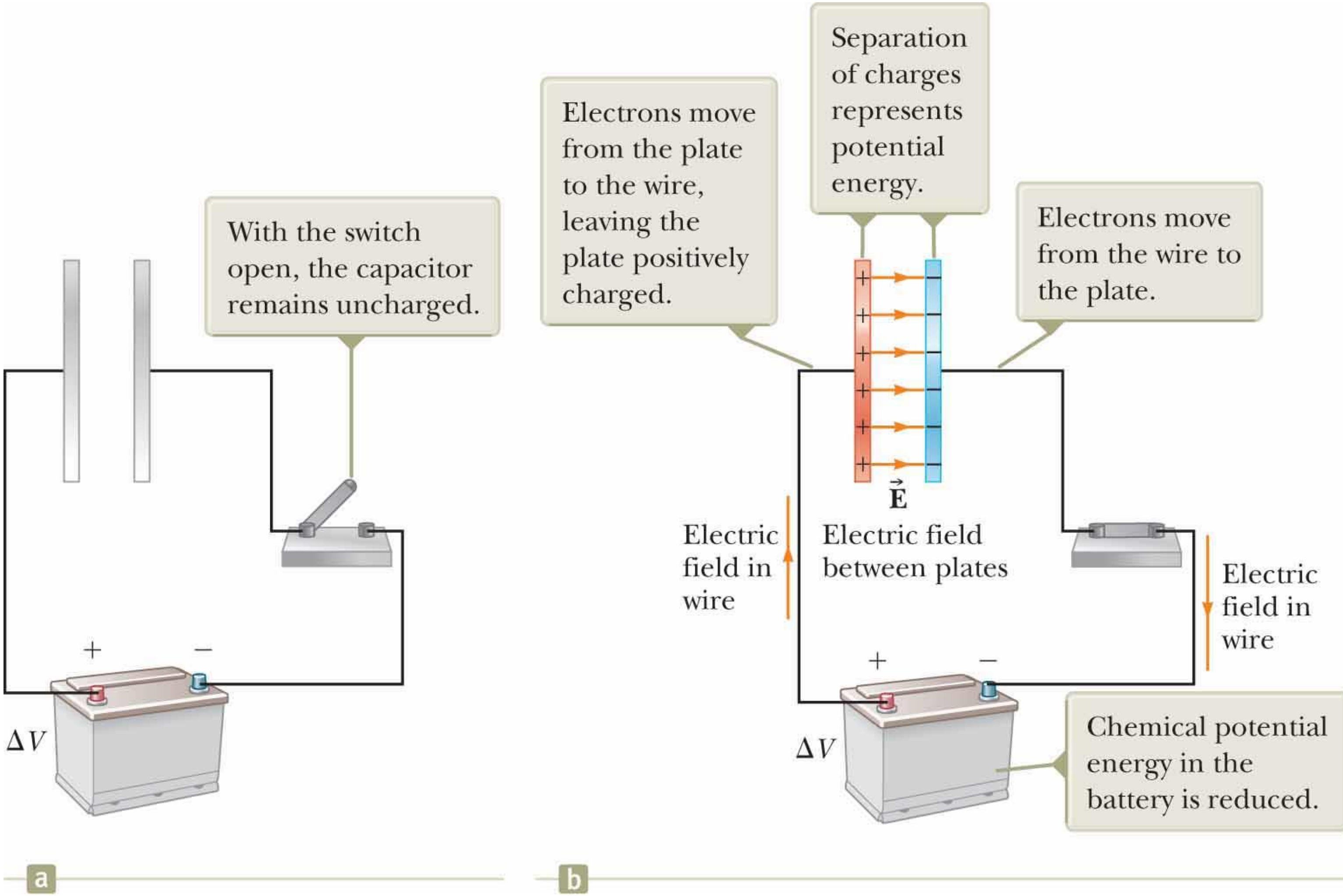
Energy Stored in a Charged Capacitor

- Because positive and negative charges separated in system of two conductors in charged capacitor: Electric potential energy stored in system

- If plates of charged capacitor connected by conductor such as wire → Charge moves between each plate and connecting wire until capacitor uncharged. Discharge often observed as visible spark

- If you accidentally touch opposite plates of charged capacitor → your fingers act as pathway for discharge. Result: electric shock – Could be dangerous if high voltages present (i.e., power supply of home theater system)
  - Because charges can be stored in capacitor even when system turned off → Unplugging system does not make it safe to open case and touch components inside

Energy Stored in a Charged Capacitor



Energy Stored in a Charged Capacitor

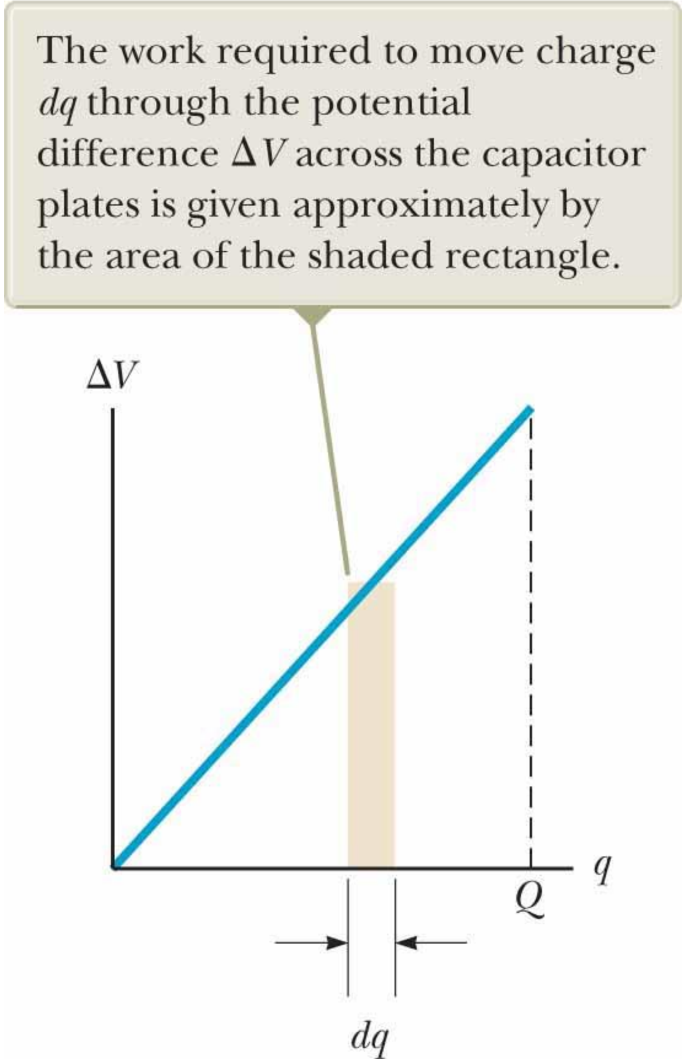
Work necessary to transfer increment of charge dq from plate carrying charge $-q$ to plate carrying charge q (at higher electric potential) is:

$$dW = \Delta V\, dq = \frac{q}{C} dq$$

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

Total energy:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$



Energy Stored in a Charged Capacitor

$$U_E = \frac{1}{2} C (\Delta V)^2$$

For parallel-plate capacitor:

- potential difference related to electric field through $\Delta V = Ed$
- capacitance: $C = \varepsilon_0 A/d$

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}(\varepsilon_0 A d) E^2$$

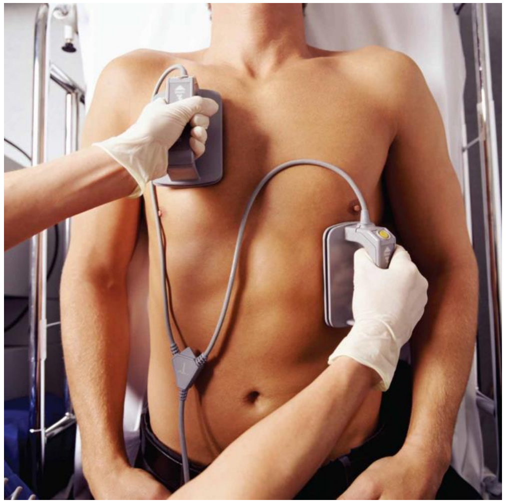
Energy per unit volume (energy density) is: $u_E = \frac{U_E}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

Expression generally valid regardless of source of electric field:

- Energy density in any electric field $\propto$ square of magnitude of electric field at given point

Portable Defibrillator

- When cardiac fibrillation (random contractions) occurs → heart produces rapid, irregular pattern of beats
  - Fast discharge of energy through heart can return the organ to normal beat pattern
- Emergency medical teams use portable defibrillators that contain batteries capable of charging capacitor to high voltage
  - Circuitry actually permits capacitor to be charged to much higher voltage than that of battery
    - Up to 360 J stored in electric field of large capacitor in defibrillator when fully charged



- Stored energy released through te heart by conducting electrodes (paddles) which are placed on both sides of victim's chest
  - Defibrillator can deliver energy to patient in ≈ 2 ms
- Paramedics must wait between applications of energy because of time interval necessary for capacitors to become fully charged

- Capacitors serve as energy reservoirs that can be slowly charged and then quickly discharged to provide large amounts of energy in a short pulse

Capacitors with Dielectrics

Dielectric: nonconducting material (i.e., rubber, glass, or waxed paper)

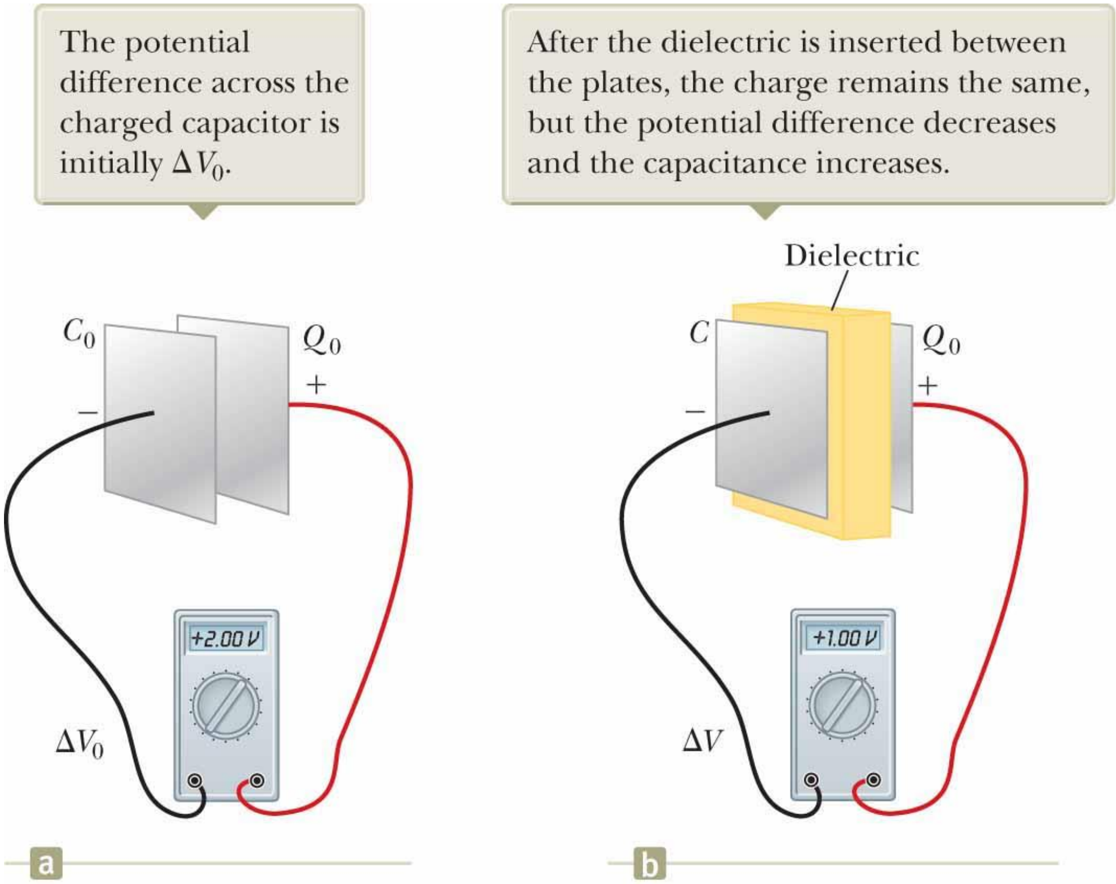
$$\Delta V = \frac{\Delta V_0}{\varepsilon}$$

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\varepsilon}$$

$$= \varepsilon \frac{Q_0}{\Delta V_0}$$

$$C = \varepsilon C_0$$

$$C = \frac{\varepsilon \varepsilon_0 A}{d}$$



ε = **dielectric constant** of the material

Capacitors with Dielectrics

$$C = \frac{\varepsilon \varepsilon_0 A}{d}$$

1) Dielectric provides:
- Increase in capacitance
- Increase in maximum operating voltage

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

2) If magnitude of electric field $E = U/d$ in dielectric exceeds dielectric strength → Insulating properties break down and dielectric begins to conduct, causing discharge.

3) When selecting capacitor for given application, must consider capacitance as well as expected voltage across capacitor in circuit.

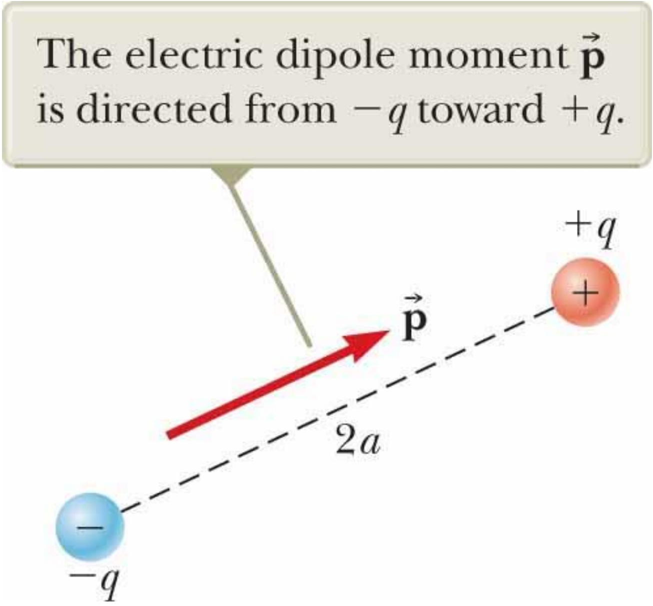
TABLE 25.1 Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polyethylene	2.30	18
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—

[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Electric Dipole in an Electric Field

Electric dipole: two charges of equal magnitude and opposite sign separated by distance *a*



Electric dipole moment: vector $\vec{p}$ directed from $-q$ toward $+q$ along line joining the charges, with magnitude

$$p \equiv 2aq$$

Electric Dipole in an Electric Field

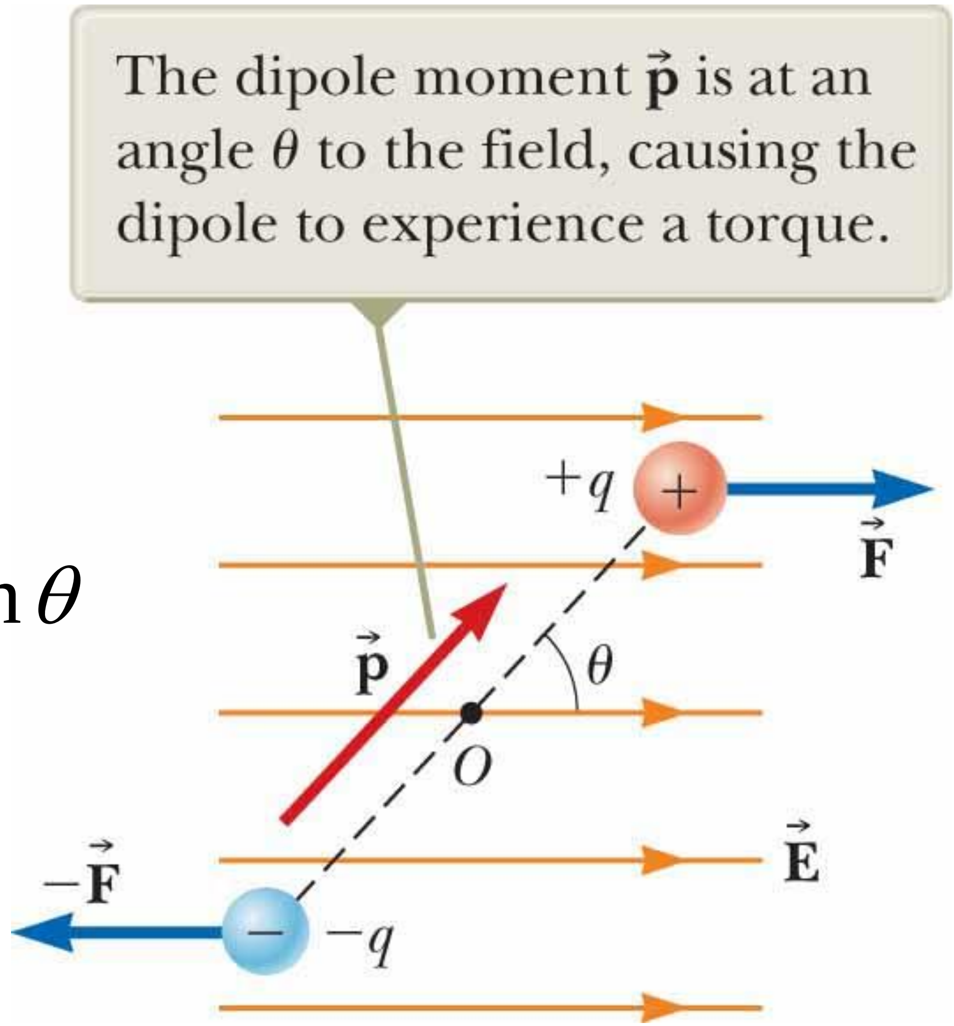
$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta$$

$$\tau_+ = \tau_- = Fa\sin\theta$$

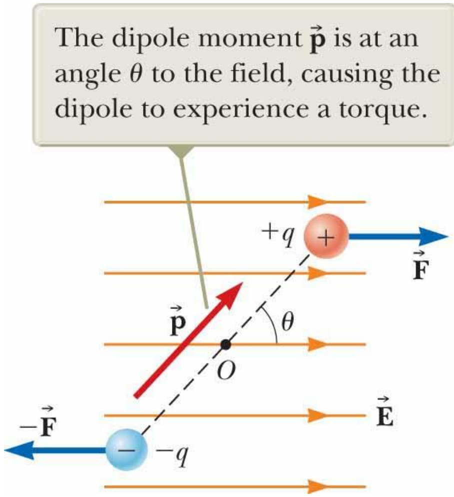
$$\tau = 2Fa\sin\theta$$

$$\tau = 2aqE\sin\theta = pE\sin\theta$$

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$



Energy of Dipole



$$dW = \tau d\theta$$

$$\tau = pE\sin\theta$$

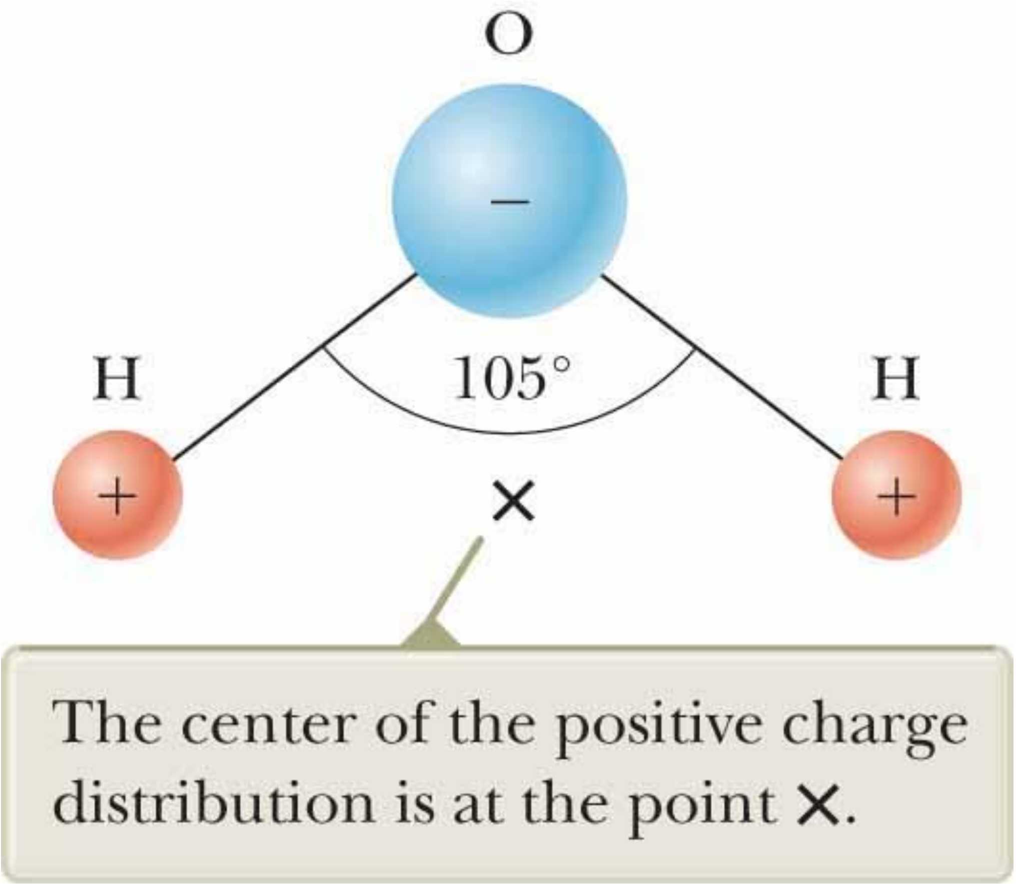
$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} pE\sin\theta\, d\theta = pE\int_{\theta_i}^{\theta_f} \sin\theta\, d\theta$$

$$= pE\left[-\cos\theta\right]_{\theta_i}^{\theta_f} = pE\left(\cos\theta_i - \cos\theta_f\right)$$

$$U_E = -pE\cos\theta$$

$$U_E = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}$$

Polar Molecules



Microwave cooking

Many molecules (such as those of water) are electric dipoles, meaning that they have a partial positive charge at one end and a partial negative charge at the other, and therefore rotate as they try to align themselves with the alternating electric field of the microwaves. Rotating molecules hit other molecules and put them into motion, thus dispersing energy. This energy, dispersed as molecular rotations, vibrations and/or translations in solids and liquids raises the temperature of the food, in a process similar to heat transfer by contact with a hotter body

Soap and the Dipole Structure of Water

- Grease and oil made up of nonpolar molecules
  - Generally not attracted to water
- Plain water not very useful for removing this type of grime

- Soap contains long molecules called *surfactants*
- In long molecule →
  - Polarity characteristics of one end of molecule can be different from those at other end
- In surfactant molecule:
  - One end acts like nonpolar molecule and other acts like polar molecule
    - Nonpolar end can attach to grease or oil molecule
    - Polar end can attach to water molecule
- Soap serves as chain → linking dirt and water molecules together
- When water rinsed away:
  - Grease and oil go with it

Induced Polarization

- Symmetric molecule (figure): no permanent polarization
- Polarization can be induced by placing molecule in electric field.