Potential Energy

The change ΔU in the potential energy

is defined to equal to the negative of the work

done by a conservative force

during the shift from an initial to a final state.

$$\Delta U = U_B - U_A = -W_{F_c} = -\int_A^B \vec{F}_c d\vec{r}$$

Potential Difference and Electric Potential

$$\Delta U_E = U_B - U_A = -W_{F_E} = -\int_A^B \mathrm{d}W_{F_E}$$

$$\mathrm{d}W_{\mathrm{F}_E} = \vec{\mathbf{F}}_E \cdot \mathrm{d}\vec{\mathbf{s}} = q\vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{s}}$$

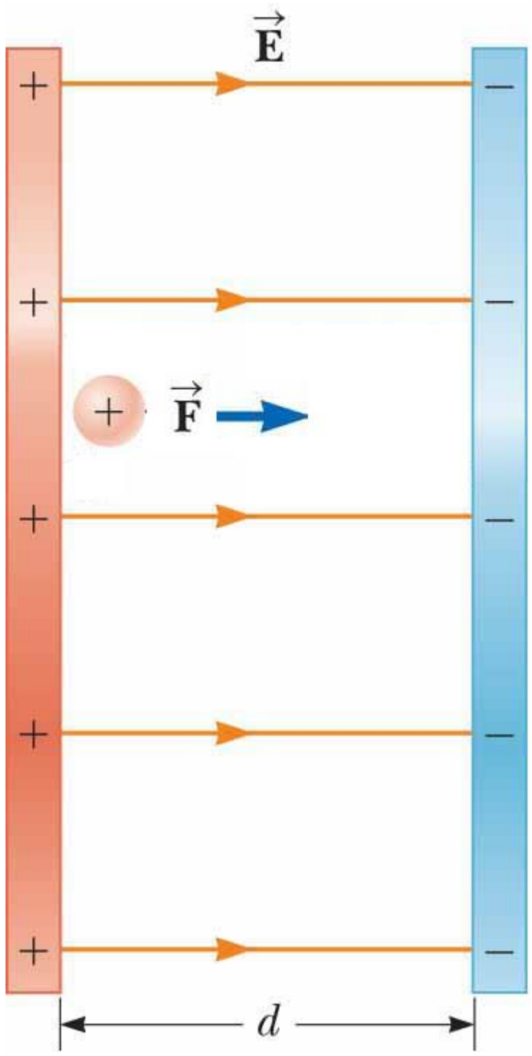
$$\Delta U_E = U_B - U_A = -q\int_A^B \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{s}}$$

Dividing potential energy by charge gives physical quantity called **electric potential** (or simply **potential**) V:

$$V = \frac{U}{q}$$

that:

- depends only on source charge distribution
- has value at every point in electric field



Potential Difference and Electric Potential

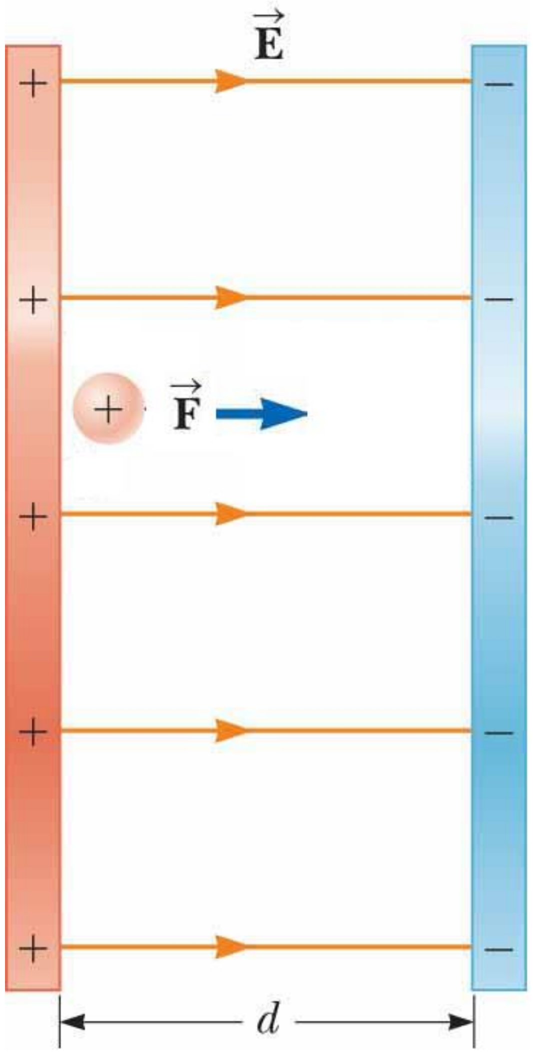
$$\Delta V = V_B - V_A = \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$W = -q\Delta V$$

$$V = \frac{U}{q} \Rightarrow \quad 1\ \mathrm{V} \equiv 1\ \mathrm{J/C}$$

$$E = \frac{F}{q} = \frac{dV}{ds} \Rightarrow \quad 1\ \mathrm{N/C} = 1\ \mathrm{V/m}$$

The electric field is a measure of the rate of change of the electric potential with respect to position.



Electronvolt

Electronvolt it is the amount of energy gained (or lost) by the charge of a single electron moving across an electric potential difference of one volt.

$$W = -q\Delta V$$

$$1\ \text{eV} = 1.60218\times10^{-19}\ \text{C}\cdot 1\ \text{V} = 1.60218\times10^{-19}\ \text{J}$$

Potential Difference in a Uniform Electric Field

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

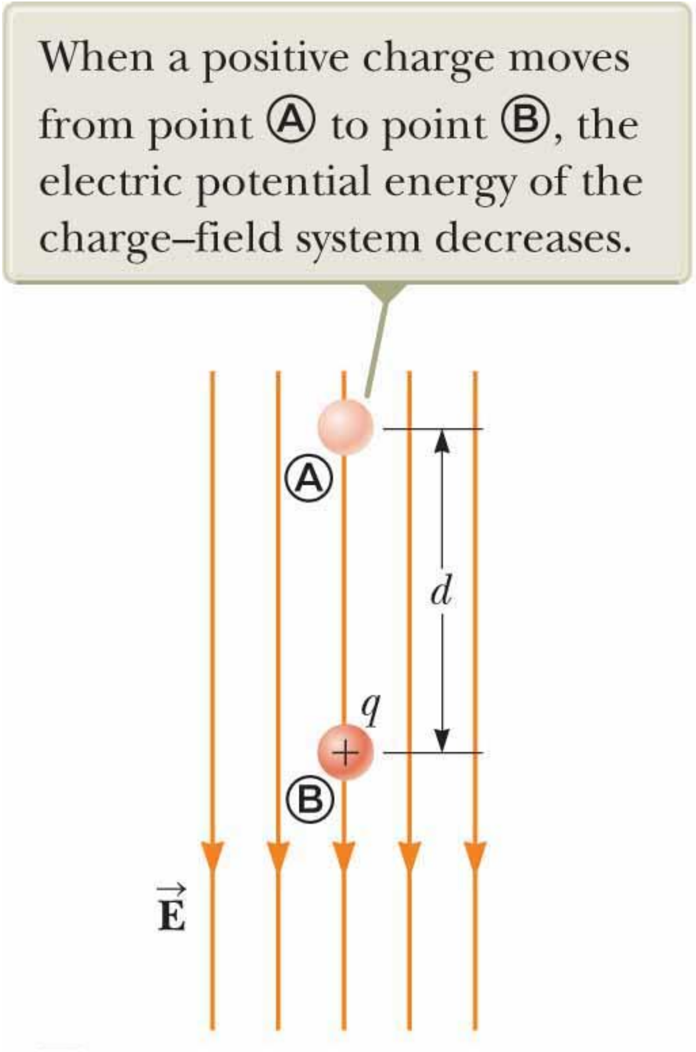
$$= -\int_A^B E\, ds\left(\cos 0°\right)$$

$$= -\int_A^B E\, ds$$

$$\Delta V = -E\int_A^B ds$$

$$\boxed{\Delta V = -Ed}$$

$$\Delta U_E = q\Delta V = -qEd$$

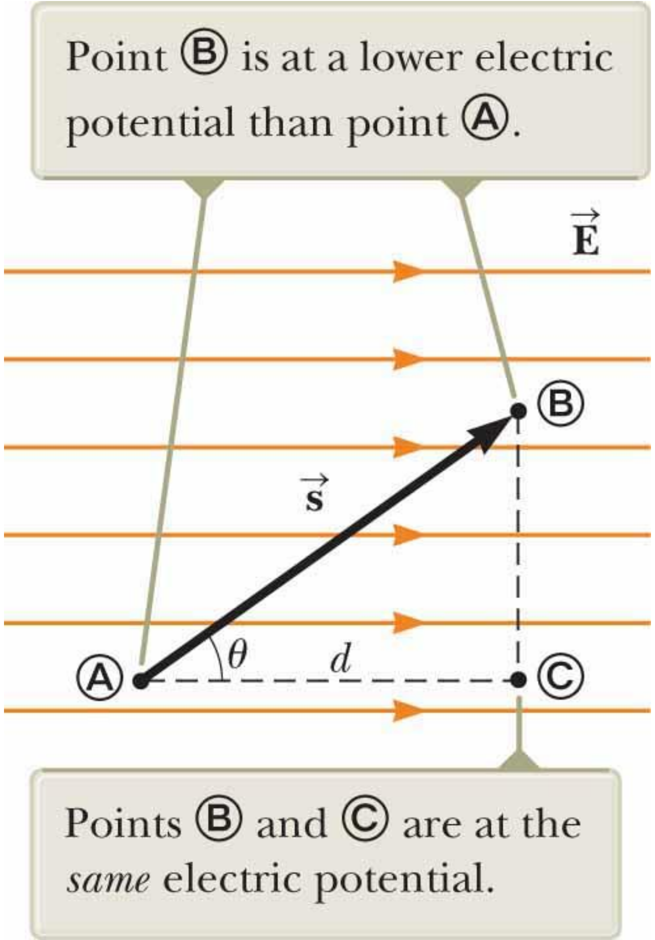


Particle in Electric Field

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\vec{\mathbf{E}} \cdot \int_A^B d\vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = \overrightarrow{\mathbf{const}}$$



$$\Delta V_{AB} = V_B - V_A = -E s_{AB} \cos(\theta) < 0$$

$$\Rightarrow V_B < V_A$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

Positive charge moves from higher to lower potential region.

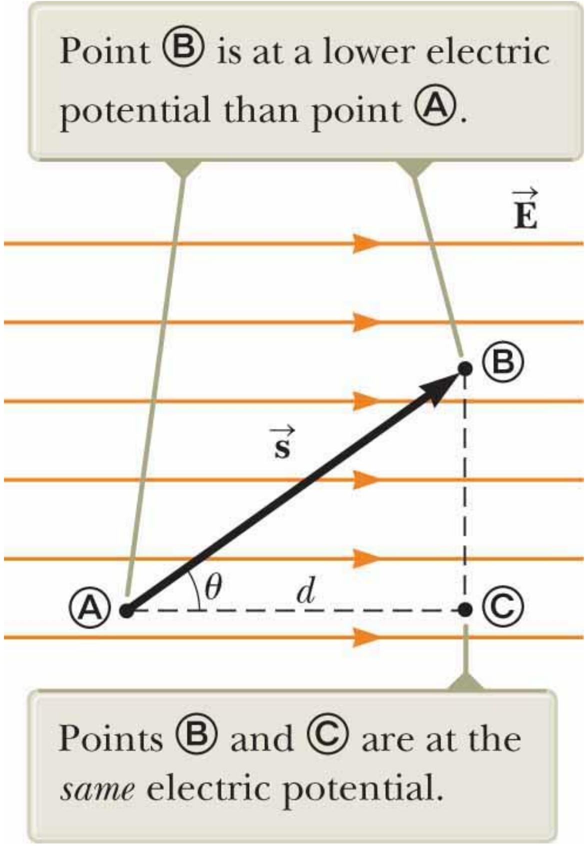
Negative charge moves from lower to higher potential region.

Equipotential Surfaces

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\vec{\mathbf{E}} \cdot \int_A^B d\vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = \overrightarrow{\mathbf{const}}$$



$$\Delta V_{CB} = V_B - V_C = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}_{CB}$$

$$= -Es_{CB}\cos\left(90^\circ\right) = 0$$

$$\Rightarrow V_B = V_C$$

Equipotential surface: any surface consisting of continuous distribution of points having same electric potential.

Equipotential surfaces are perpendicular to the field.

Electric Potential without Electric Field

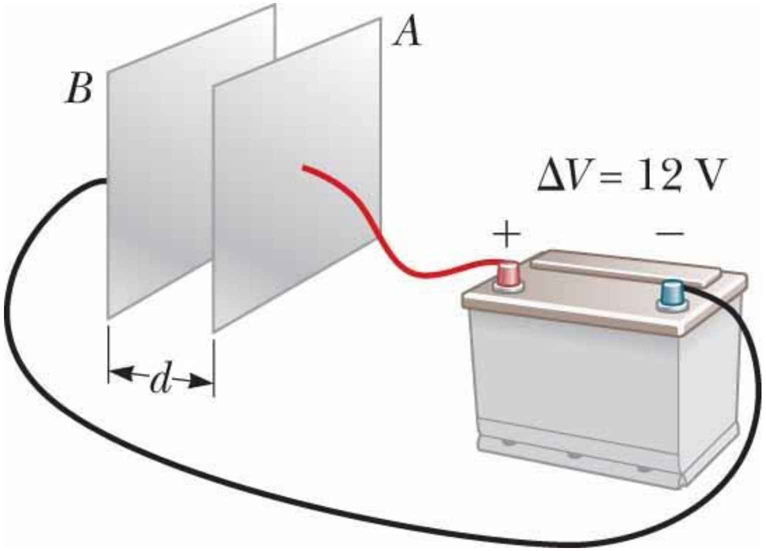
$$\Delta V = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = \vec{\mathbf{0}} \Rightarrow \Delta V = 0 \ \left(V = \mathrm{const}\right)$$

In the region without electric filed
there is no change in electric potential value.

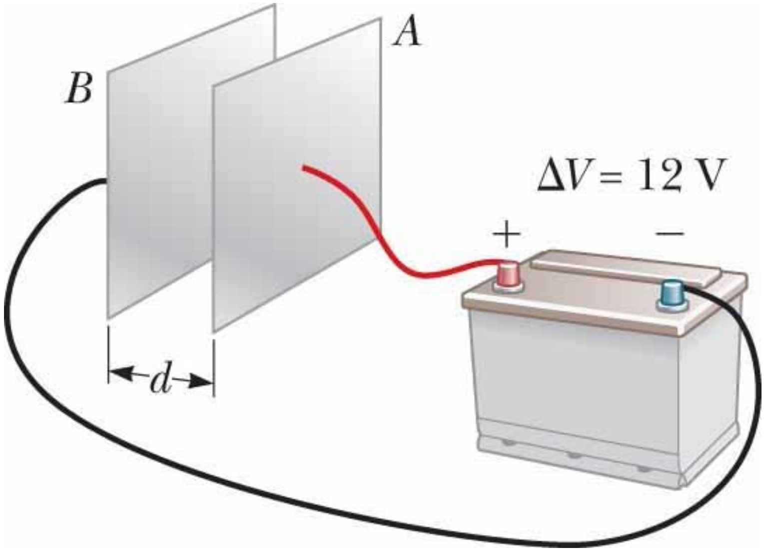
Example 24.1: The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in the figure. The separation between the plates is d = 0.30 cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.



Example 24.1: The Electric Field Between Two Parallel Plates of Opposite Charge

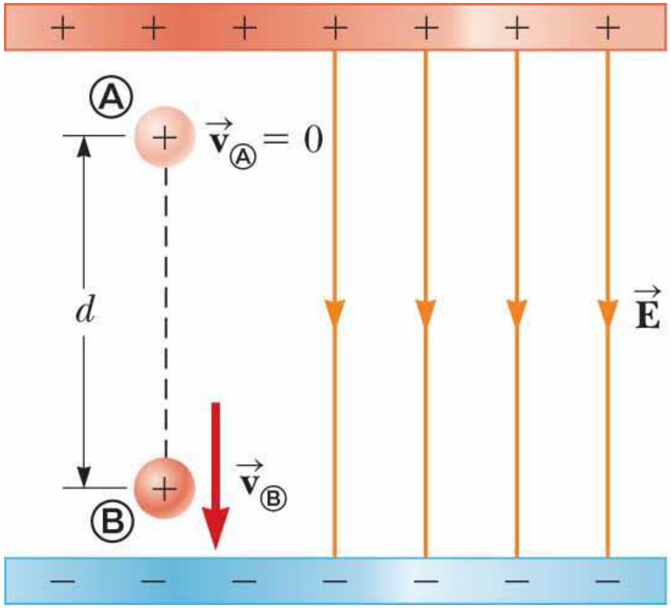
$$E = \frac{\left|V_B - V_A\right|}{d} = \frac{12\text{ V}}{0.30\times10^{-2}\text{ m}} = \boxed{4.0\times10^{3}\text{ V/m}}$$



Example 24.2:
Motion of a Proton in a Uniform Electric Field

A proton is released from rest at point A in a uniform electric field that has a magnitude of 8.0×10^4 V/m, as shown in the figure. The proton undergoes a displacement of magnitude $d = 0.50$ m to point B in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.

What will be the speed if the particle is an electron?



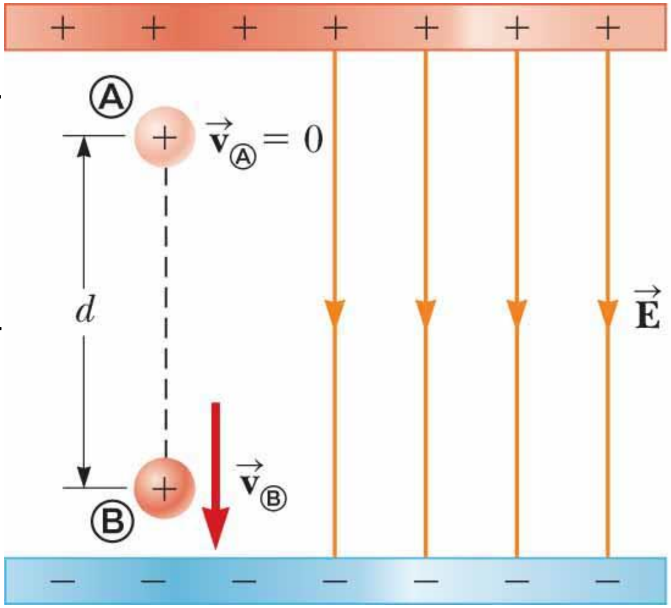
Example 24.2:
Motion of a Proton in a Uniform Electric Field

$$\Delta K + \Delta U_E = 0 \qquad \Rightarrow \left(\frac{1}{2}mv^2 - 0\right) + e\Delta V = 0$$

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v_p = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(8.0\times10^{4}\ \text{V})(0.50\ \text{m})}{1.67\times10^{-27}\ \text{kg}}} = \boxed{2.8\times10^{6}\ \text{m/s}}$$

$$v_e = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(8.0\times10^{4}\ \text{V})(0.50\ \text{m})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.2\times10^{8}\ \text{m/s}}$$



Electric Potential and Potential Energy Due to Point Charge

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

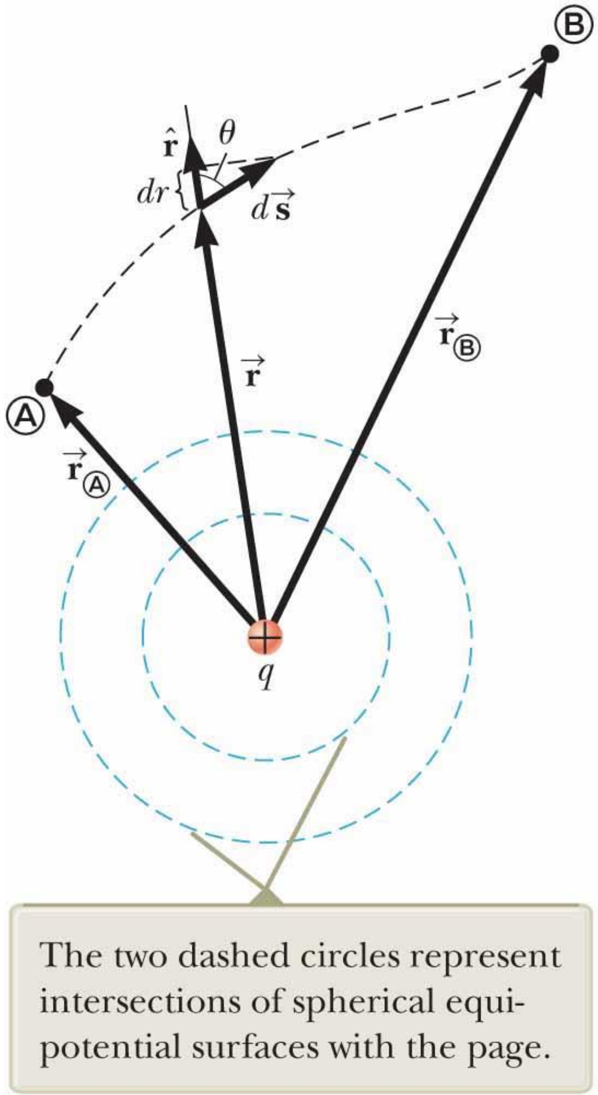
$$\vec{\mathbf{E}} = \frac{k_e q}{r^2}\hat{\mathbf{r}} \qquad \left(k_e = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2}\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

$$\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}} = 1 \cdot ds \cdot \cos\theta$$

$$ds \cos\theta = dr$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2} dr$$



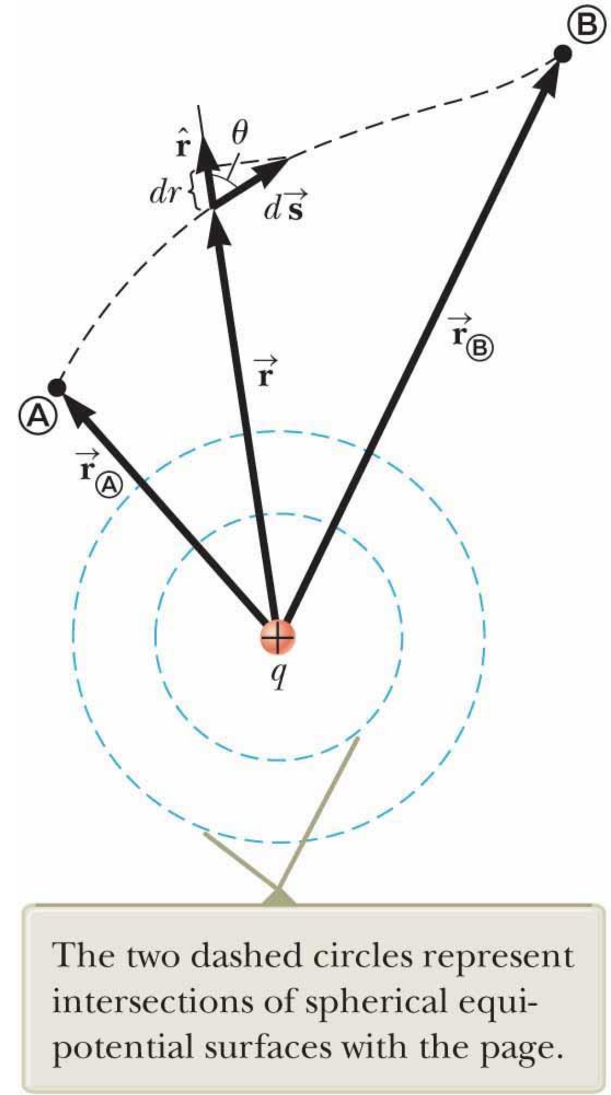
Electric Potential and Potential Energy Due to Point Charge

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e \frac{q}{r}\Big|_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

$$V_A = 0 \text{ at } r_A = \infty$$

$$V = k_e \frac{q}{r}$$



Electric Potential and Potential Energy Due to Point Charges

Electric potential resulting from two or more point charges could be obtained by applying superposition principle:

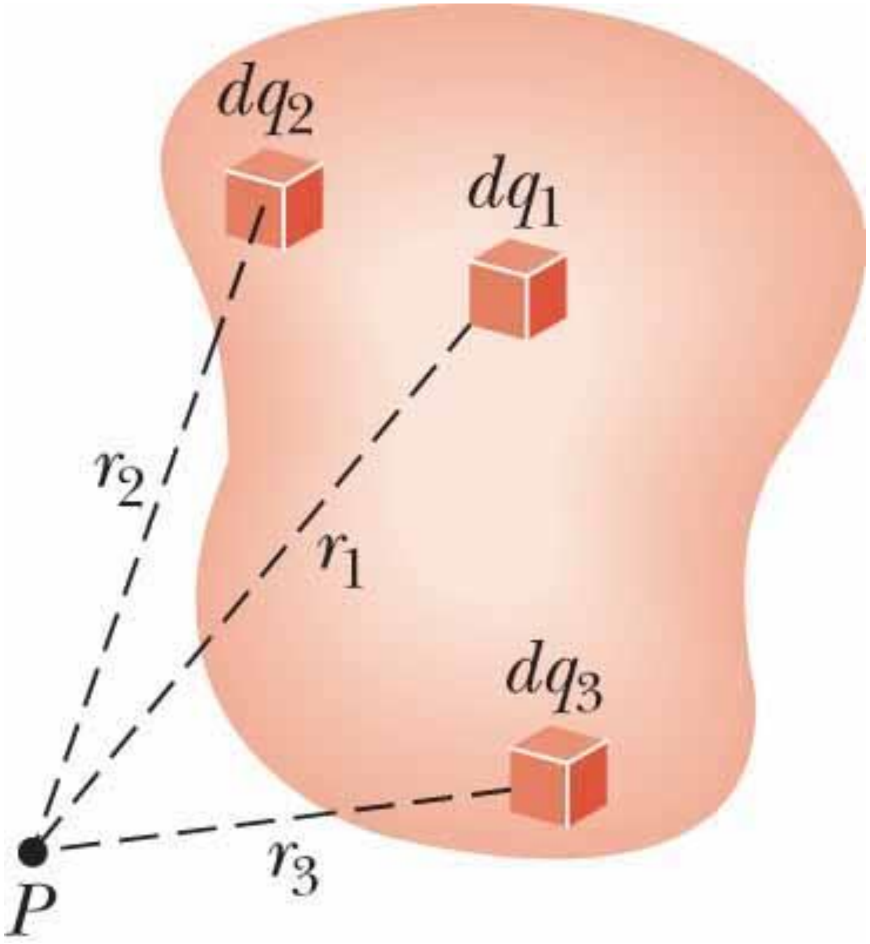
Total electric potential at some point P due to several point charges = sum of potentials due to individual charges

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric Potential Due to Continuous Charge Distributions

$$dV = k_e \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r}$$



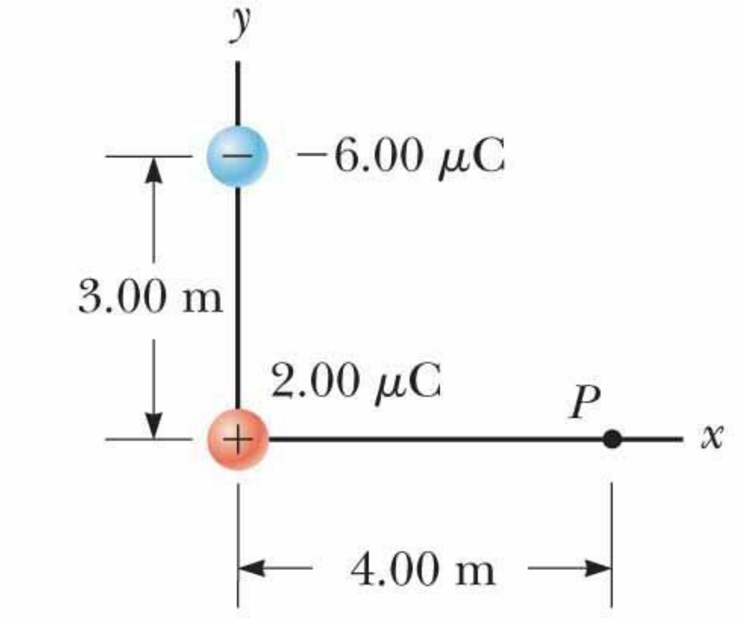
Example 24.3: The Electric Potential Due to Two Point Charges

As shown in the figure, a charge q_1 = 2.00 μC is located at the origin and a charge q_2 = –6.00 μC is located at (0, 3.00) m.

Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = \left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right) \times\left(\frac{2.00\times10^{6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00\times10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right) = \boxed{-6.29\times10^{3}\ \mathrm{V}}$$



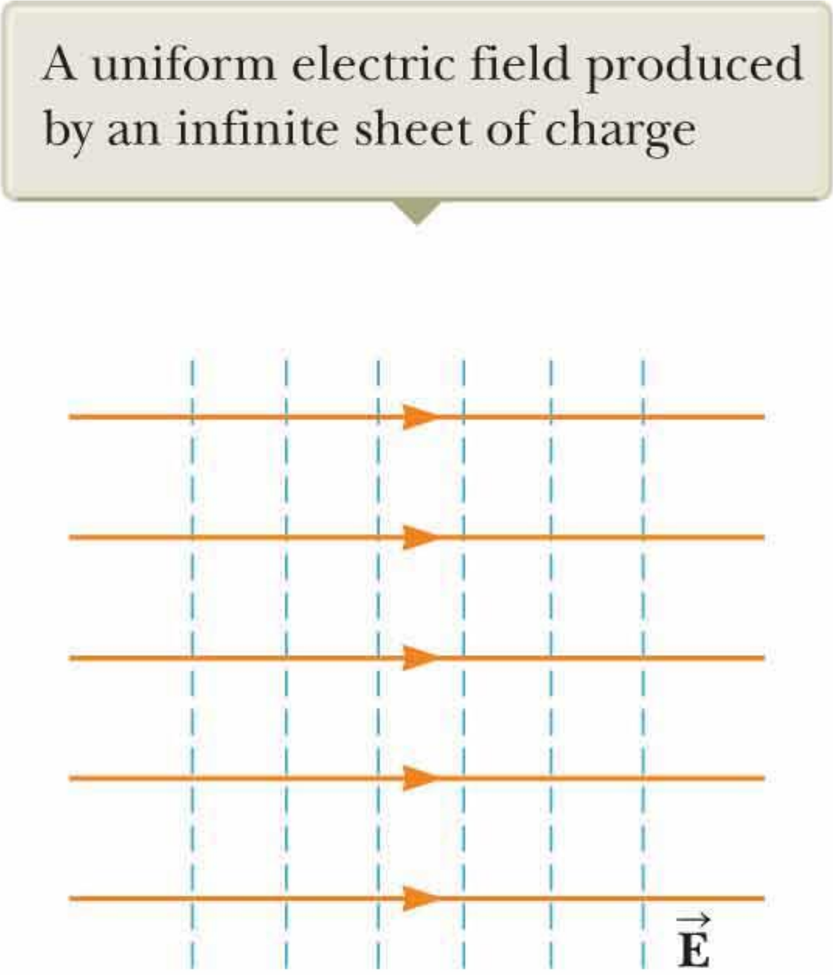
Obtaining the Value of the Electric Field from the Electric Potential

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

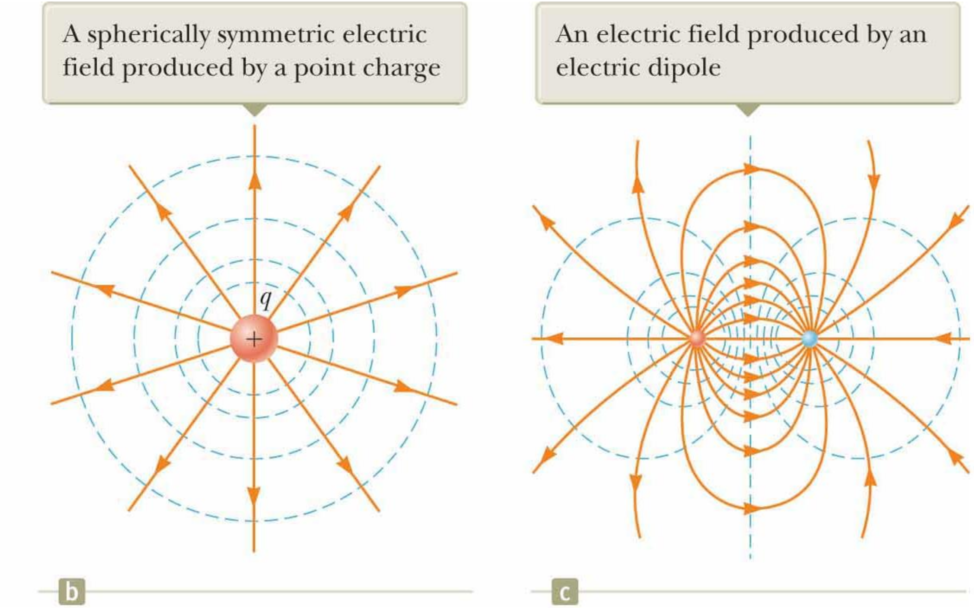
$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_x dx$$

$$E_x = -\frac{dV}{dx}$$



Obtaining the Value of the Electric Field from the Electric Potential



$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_r dr$$

$$dV = -E_r dr$$

$$E_r = -\frac{dV}{dr}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

Conductors in Electrostatic Equilibrium

1. ***E* = 0 inside conductor**

2. **Charge resides on surface of isolated conductor**

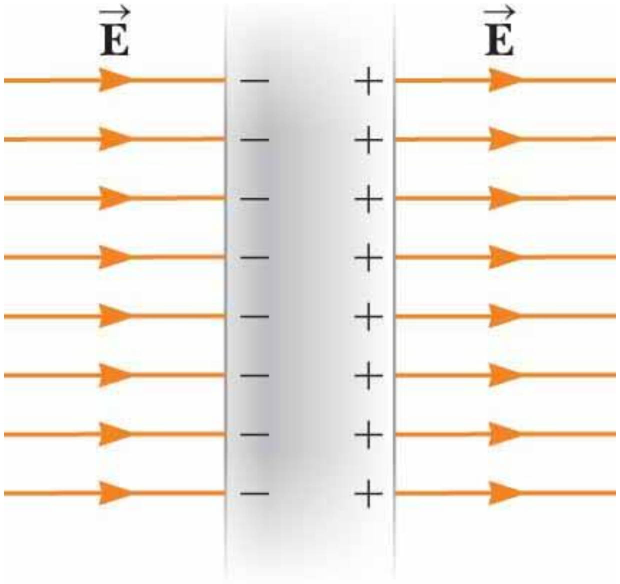
3. ***E* at point just outside conductor, perpendicular to surface, has magnitude σ/ε_0**

4. **Irregularly shaped conductor: σ greatest where radius of curvature smallest**

Conductors in Electrostatic Equilibrium

Conducting slab placed in external field $\vec{E}$:

- Before external field applied: free electrons uniformly distributed throughout conductor
- When external field applied: free electrons accelerate (to the left in figure)
    - Causing plane of negative charge to accumulate on left surface
    - Movement of electrons to left results in plane of positive charge on right surface
- These planes of charge create additional electric field inside conductor that opposes the external field
- As electrons move surface charge densities on left and right surfaces increase until magnitude of internal field = external field
- Result: net field = 0 inside conductor



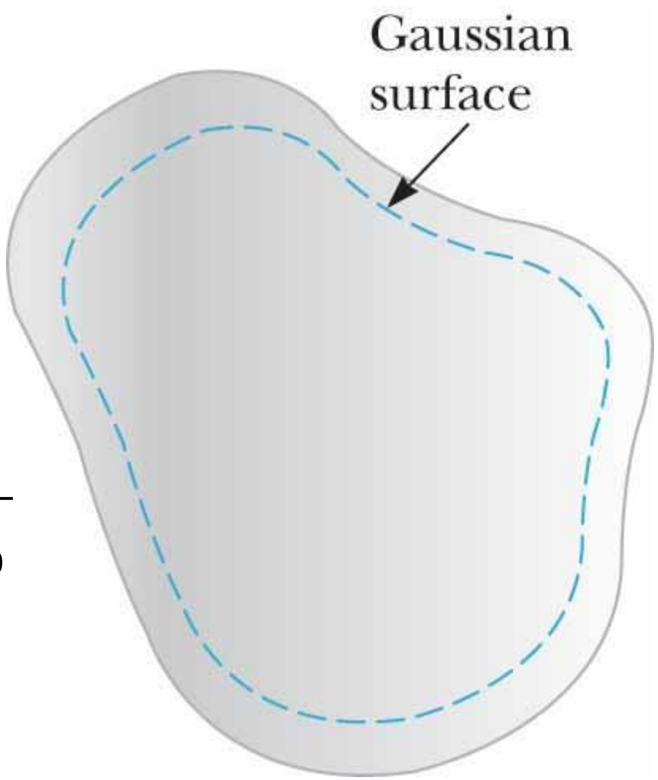
$E = 0$ inside conductor

Conductors in Electrostatic Equilibrium

- Gaussian surface drawn inside conductor
  - Can be very close to conductor's surface
- Electric field everywhere inside conductor = 0 when in electrostatic equilibrium
  - Electric field must be zero at every point on gaussian surface
  - Net flux through gaussian surface = 0
- Conclusion: net charge inside gaussian surface = 0

$$\Phi_E = \frac{q}{\varepsilon_0}$$

- No net charge inside gaussian surface (which is arbitrarily close to conductor's surface) →
  - Any net charge on conductor must reside on surface

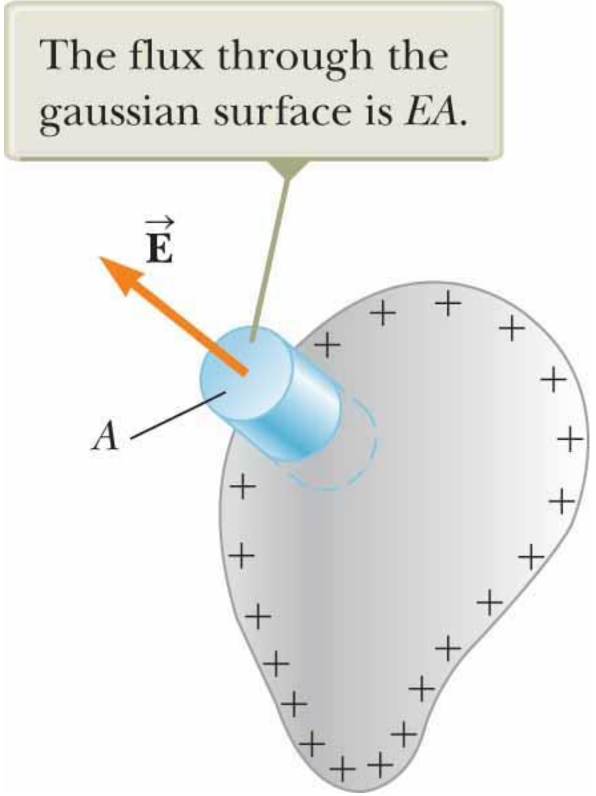


Charge resides on surface of isolated conductor

Electric Fields and Charged Conductors

$$\Phi_E = \oint E\,\mathrm{d}A = EA = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$



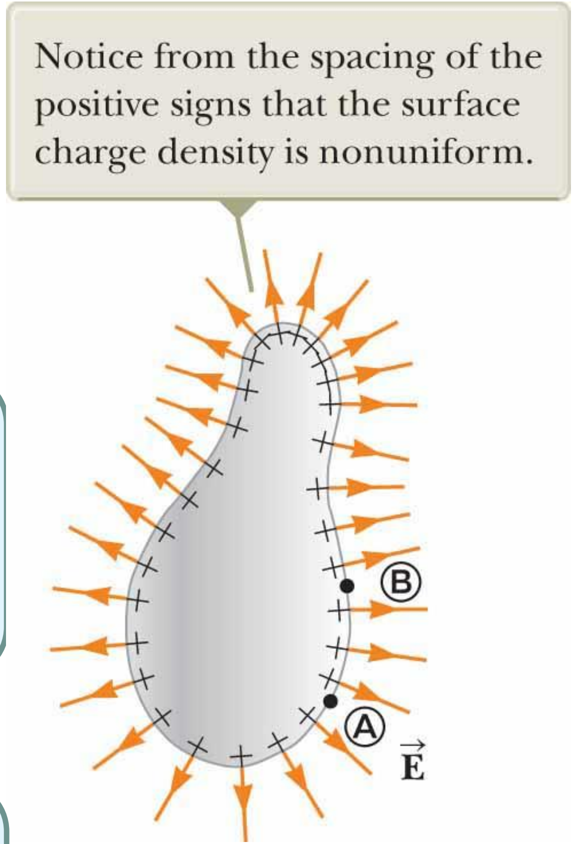
E at point just outside conductor is perpendicular to surface and has magnitude σ/ε_0

Electric Fields and Charged Conductors

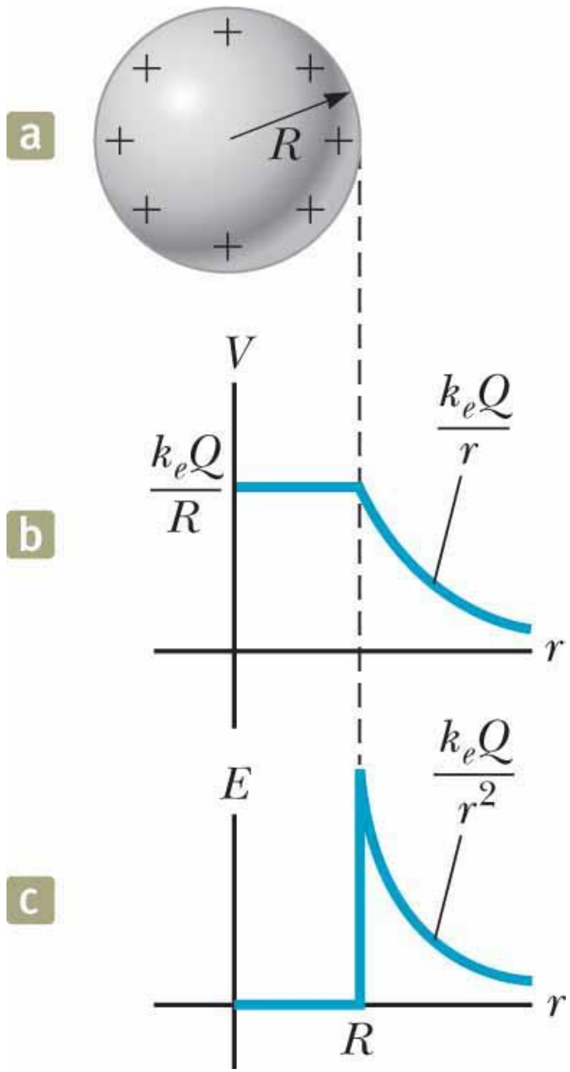
$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface: every point on the surface of a charged conductor in equilibrium is at the same electric potential.

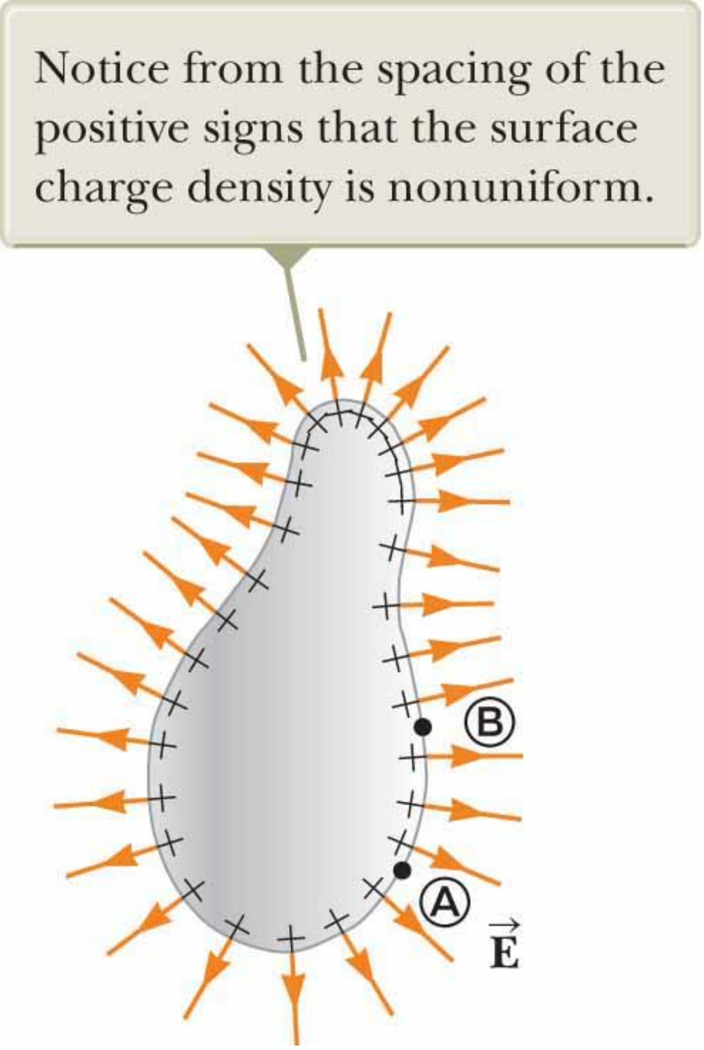
Furthermore, because the electric field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.



Electric Potential and Electric Field of Charged Conductor



Surface Charge Density on Charged Conductior



Surface Charge Density on Charged Conductor

Two spheres very far apart connected by a wire:

- Potential at surface of each sphere $V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$

equal because connecting wire assures that whole system a single conductor

- Ratio of electric fields at surfaces of two spheres

$$\frac{E_1}{E_2} = \frac{k_e \frac{q_1}{r_1^{\,2}}}{k_e \frac{q_2}{r_2^{\,2}}} = \frac{\frac{1}{r_1} V}{\frac{1}{r_2} V} = \frac{r_2}{r_1}$$

= inverse ratio of radii of spheres

- Field strong when radius small
- Field weaker when radius larger

$$r_2 < r_1 \Rightarrow E_1 < E_2$$

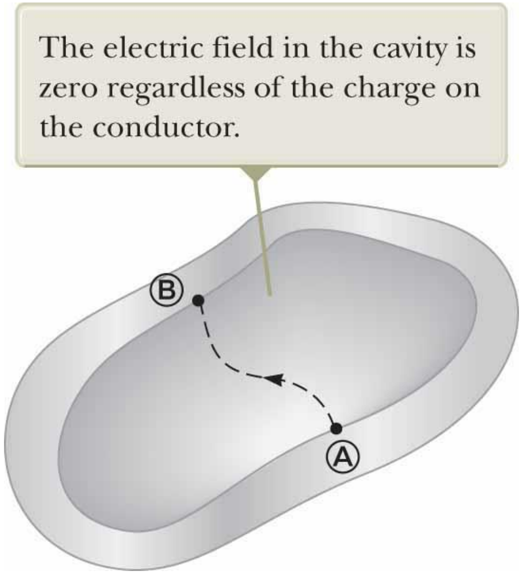
$$(\sigma = E\varepsilon_0)$$

- Electric field reaches very high values at sharp points

$$\sigma_1 < \sigma_2$$

Irregularly shaped conductor: σ greatest where radius of curvature smallest

A Cavity Within a Conductor



Conductor of arbitrary shape contains cavity

- Assume no charges inside cavity
  - Electric field inside cavity must be *zero* regardless of charge distribution on outside surface of conductor (Gauss law)
  - Field in cavity = 0 even if electric field exists outside conductor

Every point on conductor at same electric potential:
any two points A and B on cavity's surface must be at same potential

$$V_{\mathrm{B}} - V_{\mathrm{A}} = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Faraday Cage

- ***Faraday cage:*** conducting material, either solid or mesh, surrounding interior space
- Used to protect sensitive electronic equipment
- Protects you if you are inside a car during a lightning storm
  - Metal body of car acts as Faraday cage $\rightarrow$ any charge on car due to strong electric fields in car on outer surface
  - Electric field inside = 0
- Faraday cages: negative effect, i.e., loss of cellphone service inside a metal elevator