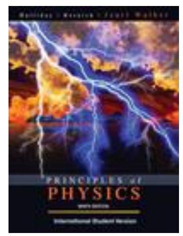
PHYSICS

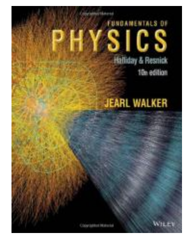
dr Bohdan Bieg (room 36A)

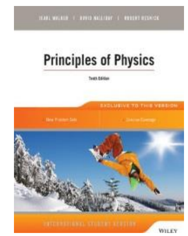
- Lectures
- Exercises
- Laboratories

Bibliography:

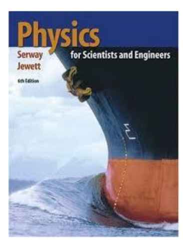
- Halliday, Resnick, Walker: **Fundamentals of Physics**

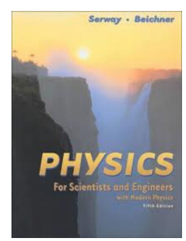


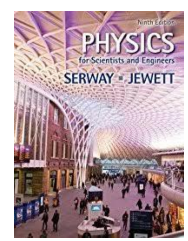




- Serway: **Physics for Scientists and Engineers**



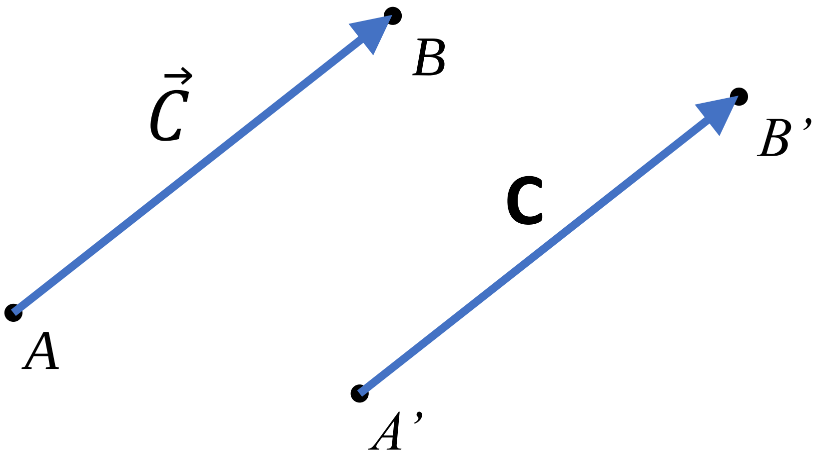




Vectors

An object that has:

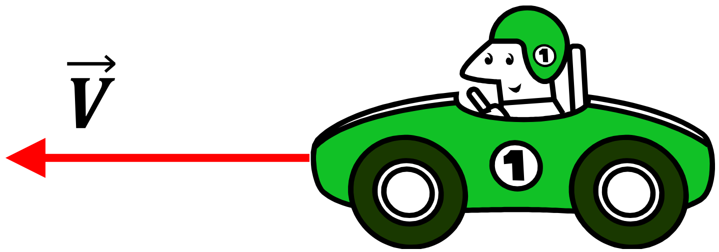
- direction
- magnitude (length)
- *unit*



Vectors

Examples:

- position - $\vec{r}$
- displacements - $\overrightarrow{\Delta r}$
- velocity - $\vec{V}$
- acceleration - $\vec{a}$



- force - $\vec{F}$
- linear momentum - $\vec{p}$
- angular momentum - $\vec{L}$
- electric field - $\vec{E}$
- electric displacement field - $\vec{D}$
- magnetic field - $\vec{B}$
- magnetic field strength - $\vec{H}$

Scalars

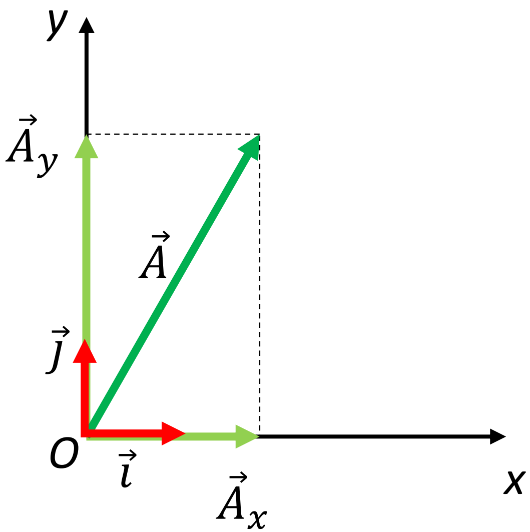
A scalar quantity is completely specified by a single value with an appropriate unit.

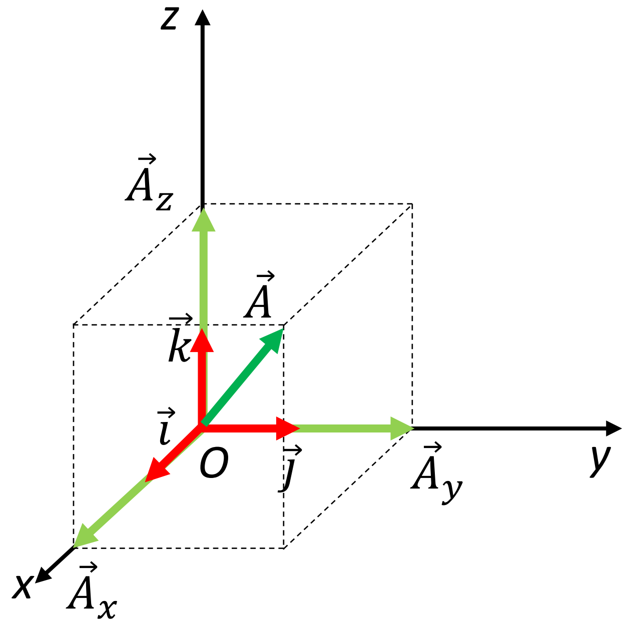
Examples:

- distance - S
- speed - V
- average speed - $\bar{V}$
- time - t
- mass - m
- rotational inertia - I
- work - W
- energy - E
- heat - Q
- charge - q
- resistance - R
- capacitance - C
- inductance - L

Coordinate systems

Cartesian coordinates (*rectangular coordinates*)





$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\vec{A}_x = A_x\vec{i} \quad \vec{A}_y = A_y\vec{j}$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = [A_x, A_y]$$

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_y$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k} = [A_x, A_y, A_z]$$

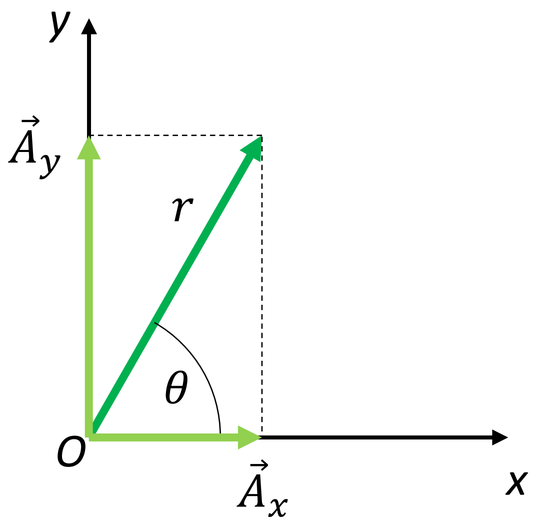
$\vec{i}, \vec{j}, \vec{k}$ - unit vectors

A_x, A_y, A_z - components of the vector

Coordinate systems

Polar coordinate system: (r, θ)

- *r* – the distance from the origin *O* to the end point
- θ – the angle between a line drawn from the origin to the end point and a fixed axis.

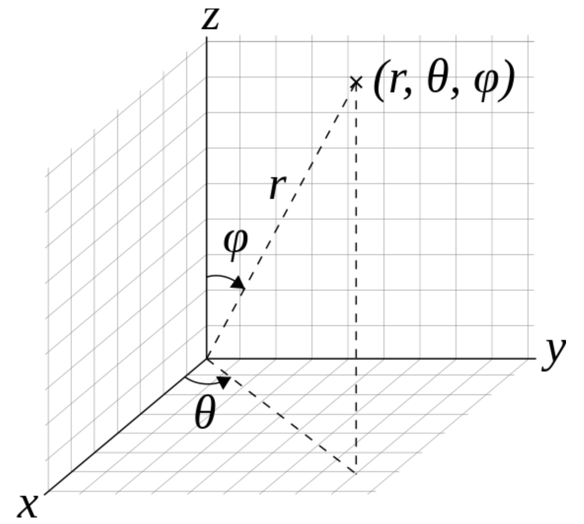


$$\begin{cases} A_x = r\cos(\theta) \\ A_y = r\sin(\theta) \end{cases}$$

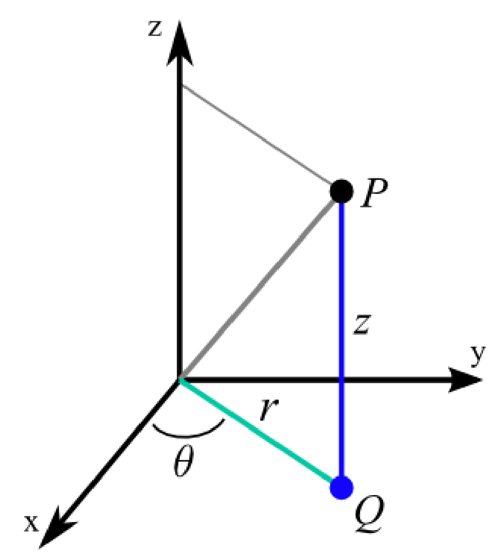
$$\begin{cases} r = \sqrt{A_x^2 + A_y^2} \\ \theta = \arctan\left(\dfrac{A_x}{A_y}\right) \end{cases}$$

Coordinate systems

Spherical coordinate system: (r, θ, φ)



Cylindrical coordinate system: (r, θ, z)

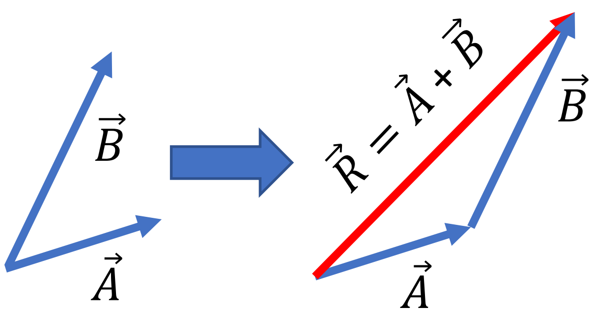


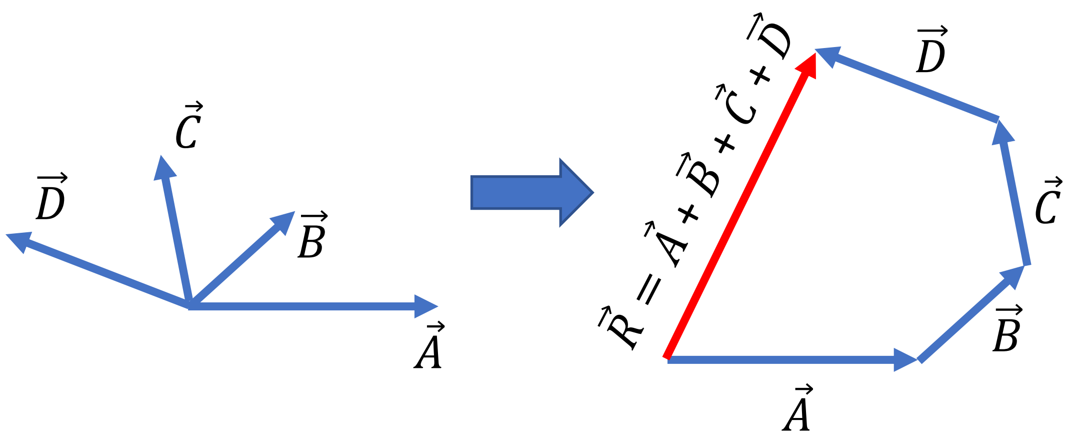
Adding Vectors

$$\overrightarrow{R} = \overrightarrow{A} + \overrightarrow{B}$$

Graphical method:

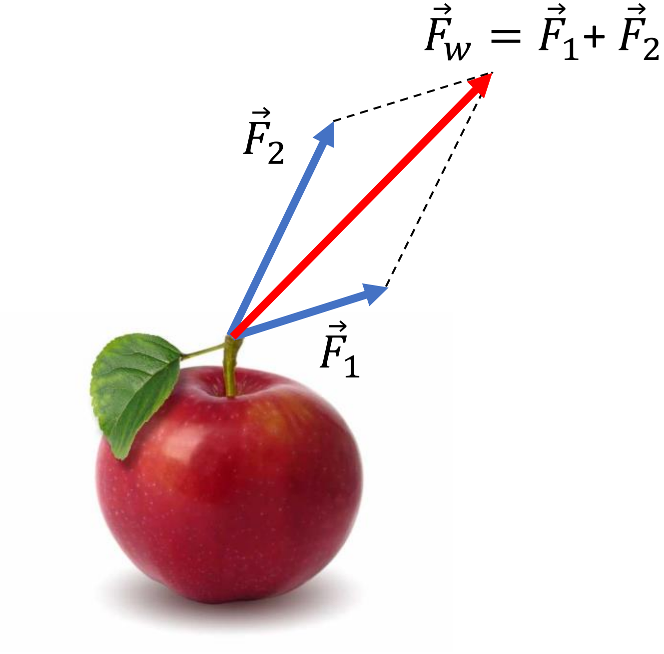
To add vector $\vec{B}$ to vector $\vec{A}$, first draw vector $\vec{A}$, with its magnitude represented by a convenient length scale, and then draw vector $\vec{B}$ to the same scale with its tail starting from the tip of $\vec{A}$. The resultant vector is the vector drawn from the tail of $\vec{A}$ to the tip of $\vec{B}$.





Adding Vectors

$$\overrightarrow{R} = \overrightarrow{A} + \overrightarrow{B}$$



Example:

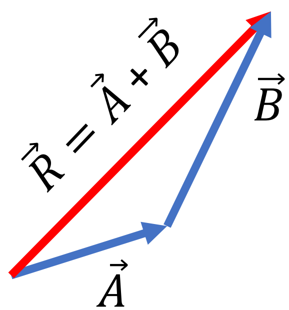
- net force: $\vec{F}_w = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$

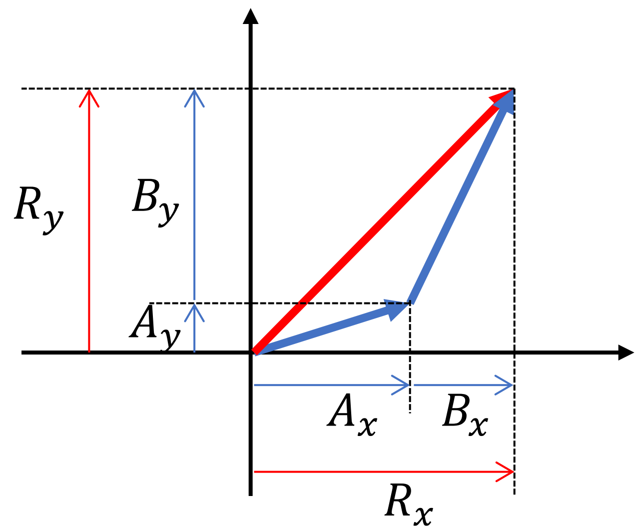
- net electric field: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$

Adding Vectors

Algebraic method:

To add vector $\vec{B}$ to vector $\vec{A}$, find the sum of both vectors corresponding components.



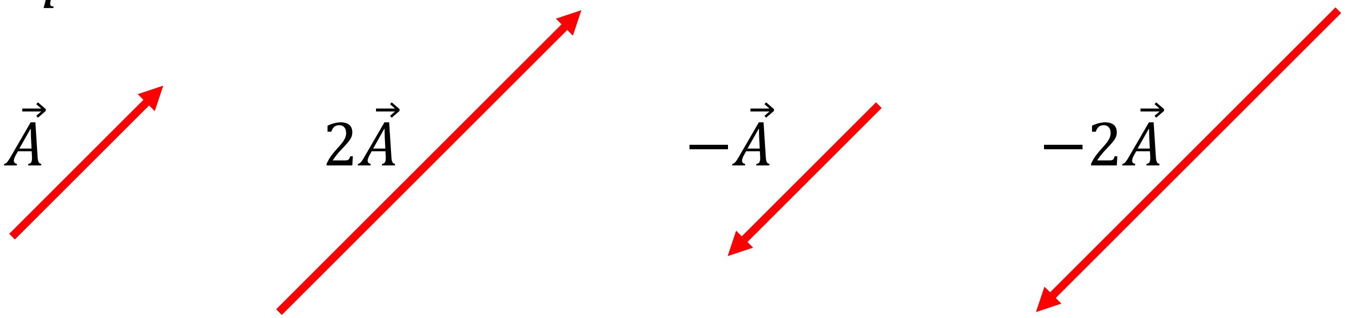


$$\vec{R} = [R_x, R_y, R_z] = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j} + (A_z + B_z)\vec{k}$$

Multiplying a Vector by a Scalar

If vector $\vec{A}$ is multiplied by a positive scalar quantity q, then the product $\vec{R} = q \cdot \vec{A}$ is a vector that has the same direction as $\vec{A}$ and magnitude qA.

If vector $\vec{A}$ is multiplied by a negative scalar quantity q, then the product $\vec{R} = q \cdot \overrightarrow{A}$ has opposite direction to $\vec{A}$ and magnitude qA.

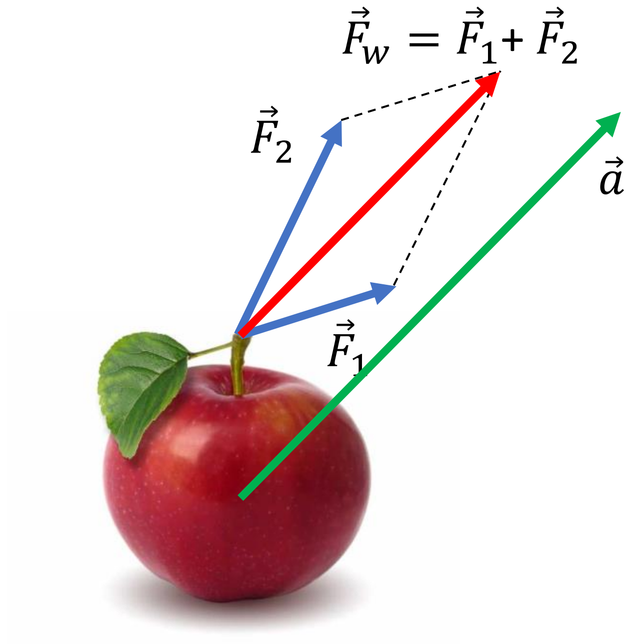


$$\vec{R} = q\vec{A} = qA_x\vec{i} + qA_y\vec{j} + qA_z\vec{k} = [qA_x, qA_y, qA_z]$$

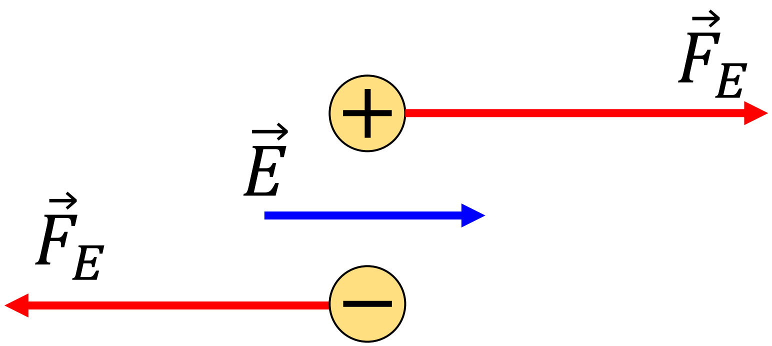
Multiplying a Vector by a Scalar

Example:

- Newton's second law: $\vec{a} = \frac{1}{m}\vec{F}_w$,



- the electric force acting on a positive or negative charge: $\vec{F}_E = q\vec{E}$

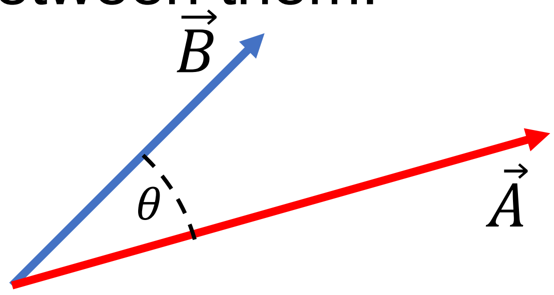


The Scalar Product (dot product)

$$R = \overrightarrow{A} \cdot \overrightarrow{B}$$

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

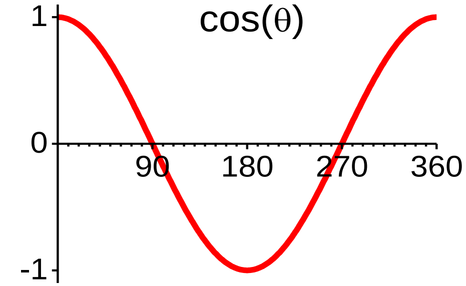
$$R = \vec{A} \cdot \vec{B} = AB\cos(\theta)$$



$$R = \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

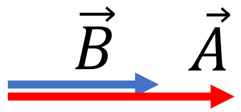
The Scalar Product – observations

$$R = \vec{A} \cdot \vec{B} = AB\cos(\theta)$$

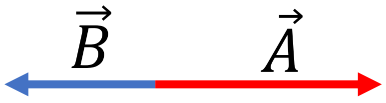


1) For two parallel vectors $(\theta = 0°)$: $R = AB$

$$\left(\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1\right)$$

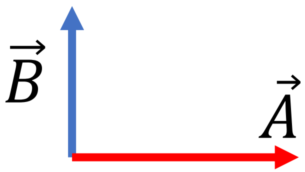


2) For two opposite vectors $(\theta = 180°)$: $R = -AB$



3) For two perpendicular vectors $(\theta = 90°)$: $R = 0$

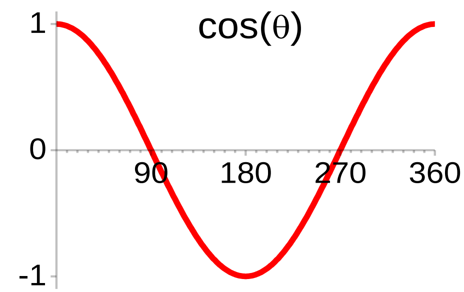
$$\left(\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 0\right)$$



$$R = \vec{A} \cdot \vec{B} = \left(A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\right) \cdot \left(B_x\vec{i} + B_y\vec{j} + B_z\vec{k}\right)$$
$$= A_xB_x + A_yB_y + A_zB_z$$

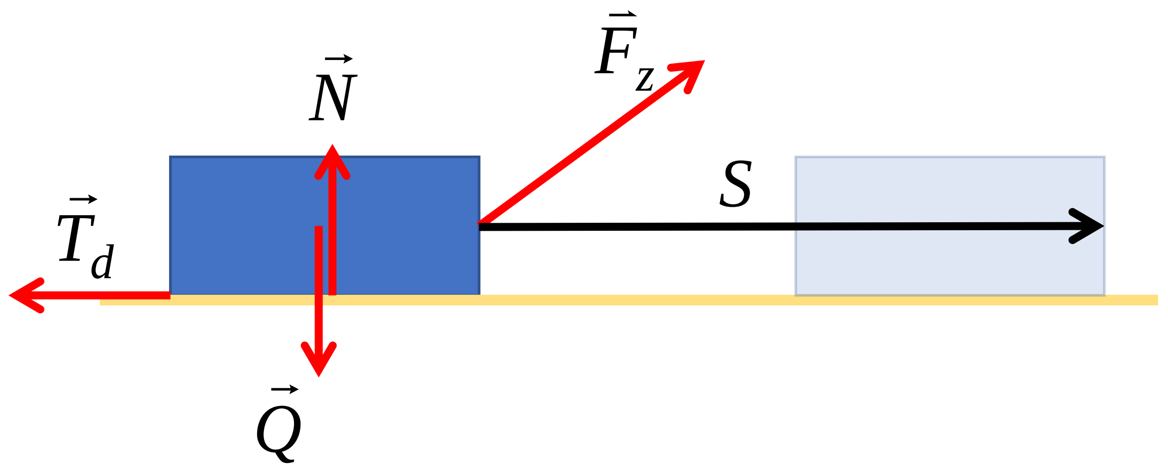
The Scalar Product (dot product)

$$R = \overrightarrow{A} \cdot \overrightarrow{B}$$



Example:

- work: $W = \vec{F} \cdot \vec{S}$,



$$W_{F_z} > 0$$

$$W_Q = 0$$

$$W_N = 0$$

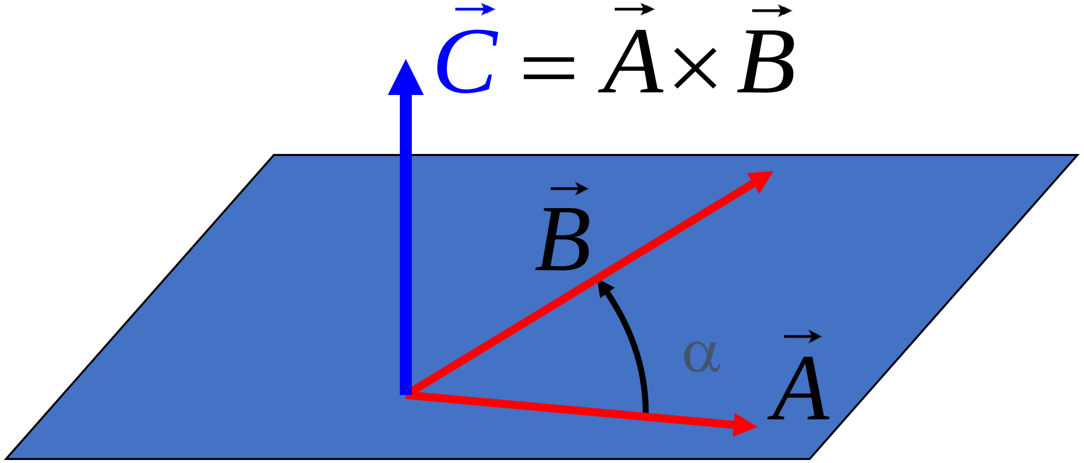
$$W_{T_d} < 0$$

The Vector Product (cross product):

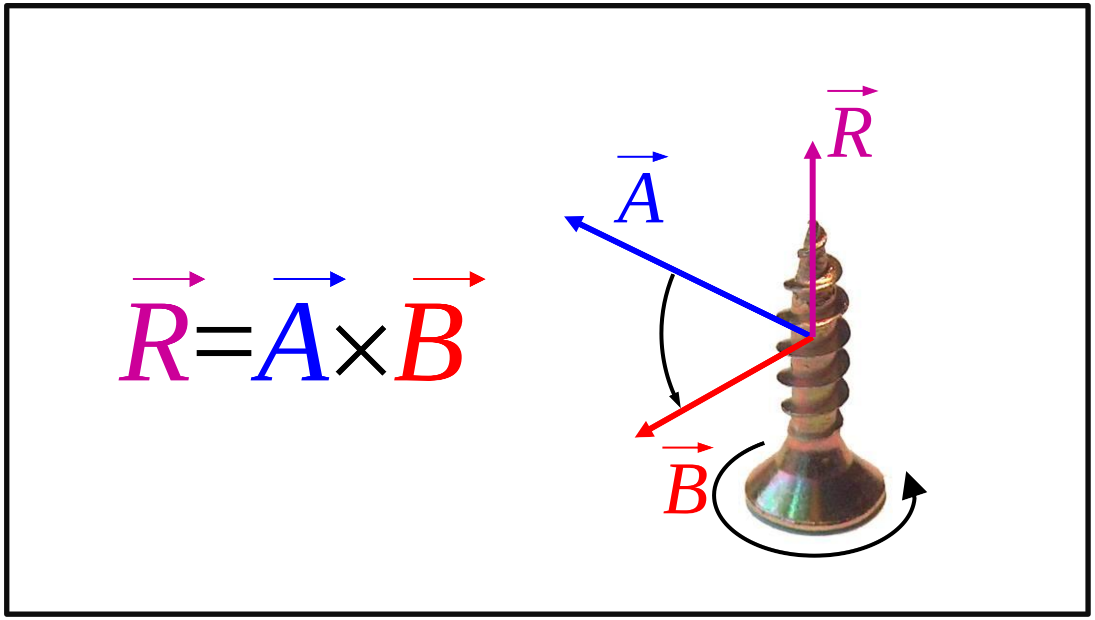
$$\vec{R} = \vec{A} \times \vec{B}$$

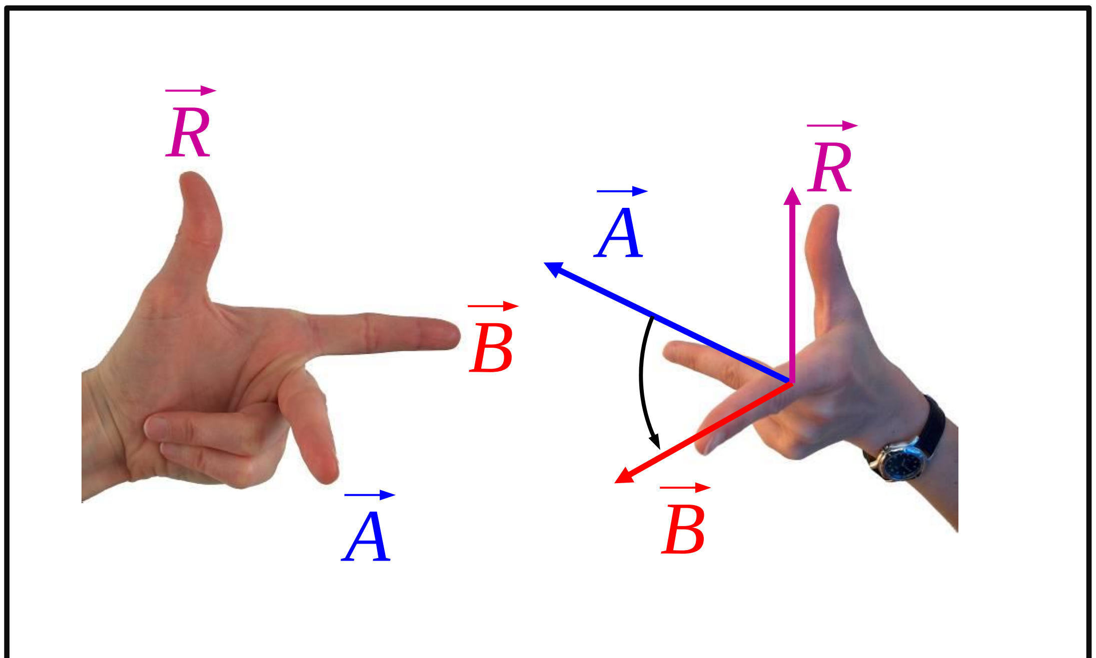
The vector product of any two vectors $\vec{A}$ and $\vec{B}$ is a vector with:

- the magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle θ between them: $R = AB\sin(\theta)$
- the direction perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, and this direction is determined by the screw rule or the right-hand rule or the left-hand rule.

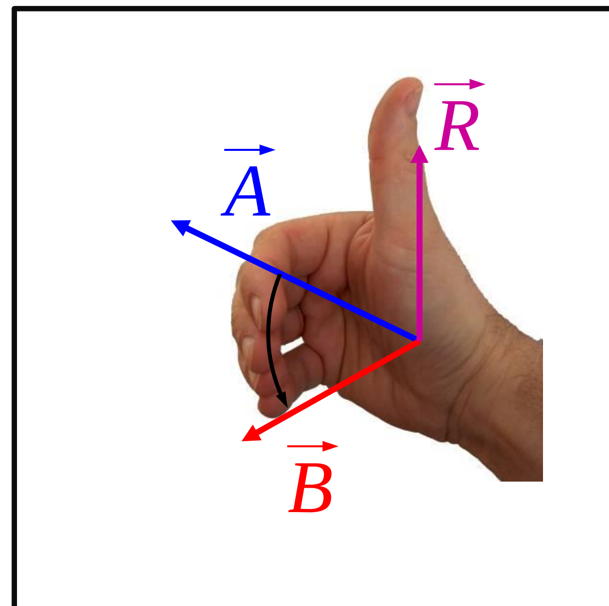


Screw rule:





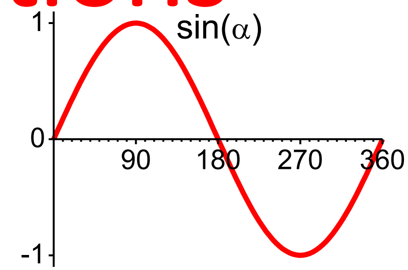
Left-hand rule:



Right-hand rule:

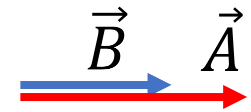
The Vector Product – observations

$$R = \vec{A} \times \vec{B} = AB\sin(\theta)$$



1) For two parallel vectors ($\theta = 0°$): $R = 0$

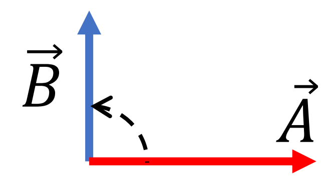
$$\left(\vec{\imath} \times \vec{\imath} = \vec{\jmath} \times \vec{\jmath} = \vec{k} \times \vec{k} = \vec{0}\right)$$



2) For two opposite vectors ($\theta = 180°$): $R = \vec{0}$

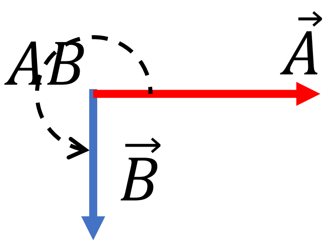
3) For two perpendicular vectors ($\theta = 90°$): $R = AB$

$$\left(\vec{\imath} \times \vec{\jmath} = \vec{k} \quad \vec{\jmath} \times \vec{k} = \vec{\imath} \quad \vec{k} \times \vec{\imath} = \vec{\jmath}\right)$$



4) For two perpendicular vectors ($\theta = 270°$): $R = -AB$

$$\left(\vec{\jmath} \times \vec{\imath} = -\vec{k} \quad \vec{k} \times \vec{\jmath} = \overrightarrow{-i} \quad \vec{\imath} \times \vec{k} = -\vec{\jmath}\right)$$



the angle θ is from $\vec{A}$ to $\vec{B}$ – always in anticlockwise direction!!!

The Vector Product – observations

$$R = \vec{A} \times \vec{B} = \left(A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\right) \times \left(B_x\vec{i} + B_y\vec{j} + B_z\vec{k}\right)$$

$$\left(\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = \vec{0}\right)$$

$$\left(\vec{i} \times \vec{j} = \vec{k} \quad \vec{j} \times \vec{k} = \vec{i} \quad \vec{k} \times \vec{i} = \vec{j}\right)$$

$$\left(\vec{j} \times \vec{i} = -\vec{k} \quad \vec{k} \times \vec{j} = \overrightarrow{-i} \quad \vec{i} \times \vec{k} = -\vec{j}\right)$$

$$R = \left(A_yB_z - A_zB_y\right)\vec{i} + (A_zB_x - A_xB_z)\vec{j} + \left(A_xB_y - A_yB_x\right)\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

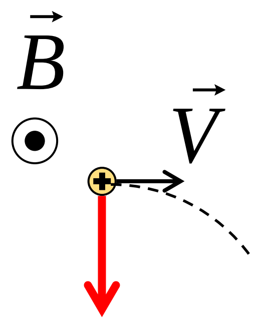
The Vector Product (cross product):

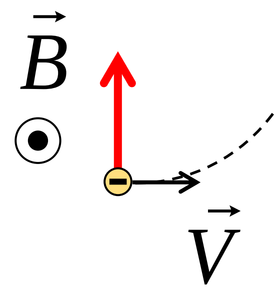
$$\vec{R} = \vec{A} \times \vec{B}$$

Example:

- torque: $\vec{M} = \vec{r} \times \vec{F}$,
- angular momentum: $\vec{L} = \vec{r} \times \vec{p}$
- the electric force acting on a moving charge: $\vec{F}_B = q\vec{V} \times \vec{B}$

$$\vec{F}_B = q\vec{V} \times \vec{B}$$





Problem:

For the two vectors $\vec{A} = 2\vec{\imath} + 3\vec{\jmath} - \vec{k}$ and $\vec{B} = -3\vec{\imath} + 2\vec{\jmath}$ determine:

a) magnitude A and B

b) sum $\vec{A} + \vec{B}$

c) difference $\vec{A} - \vec{B}$

d) dot product $\vec{A} \cdot \vec{B}$

e) cross product $\vec{A} \times \vec{B}$

f) angle between $\vec{A}$ and $\vec{B}$

g) angle between $\vec{A}$ and $\vec{A} \times \vec{B}$