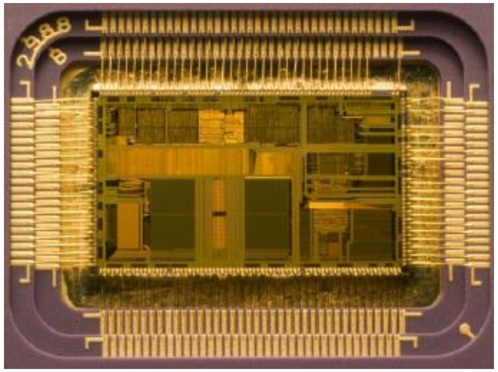
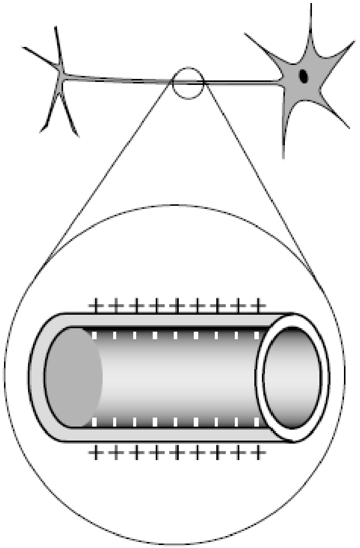
Electricity





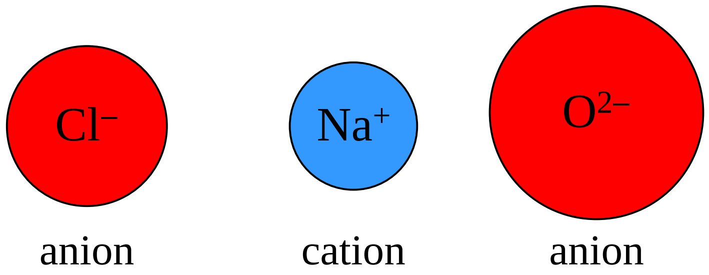
Electric Charges

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

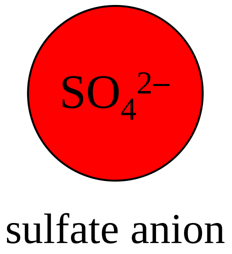
Subatomic particles

Ions – electrically charged

atoms

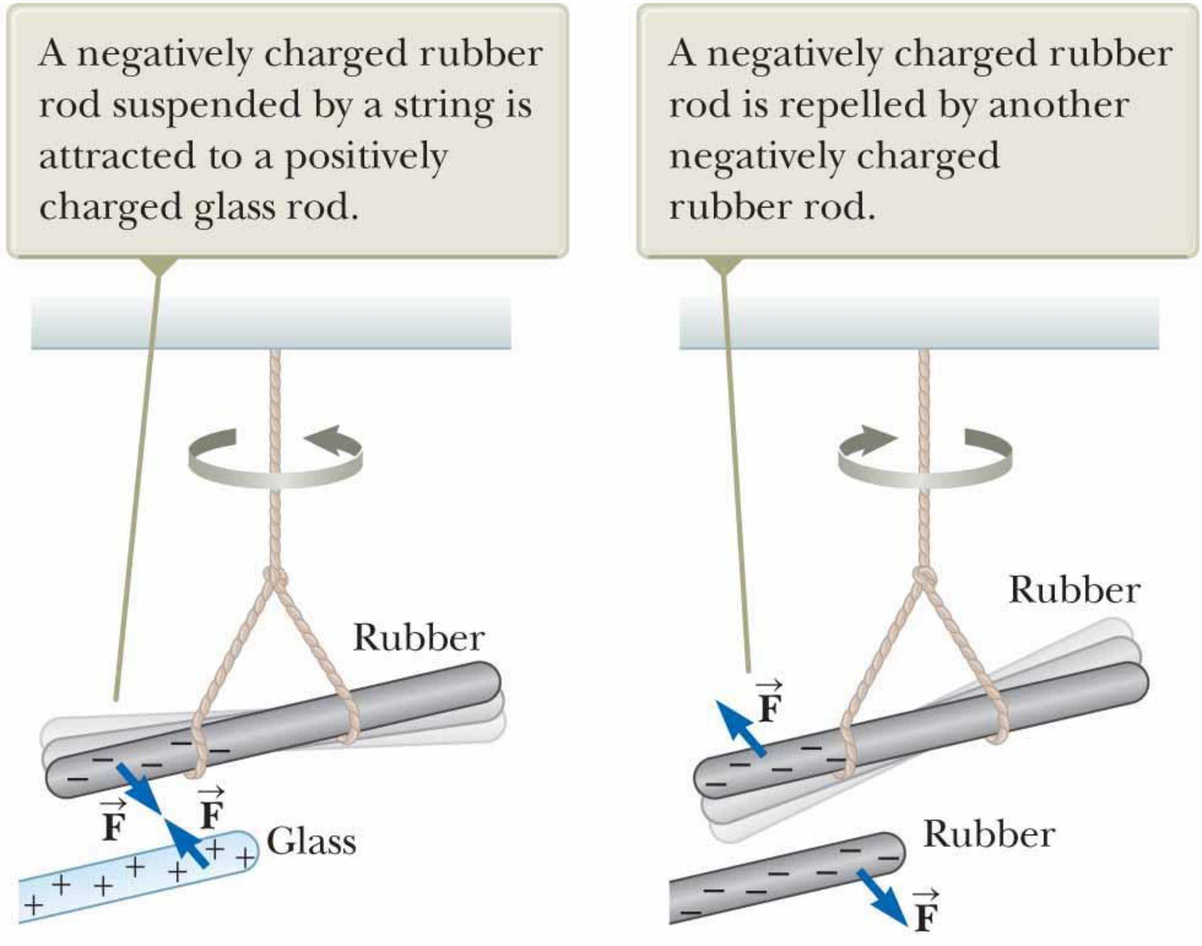


and molecules



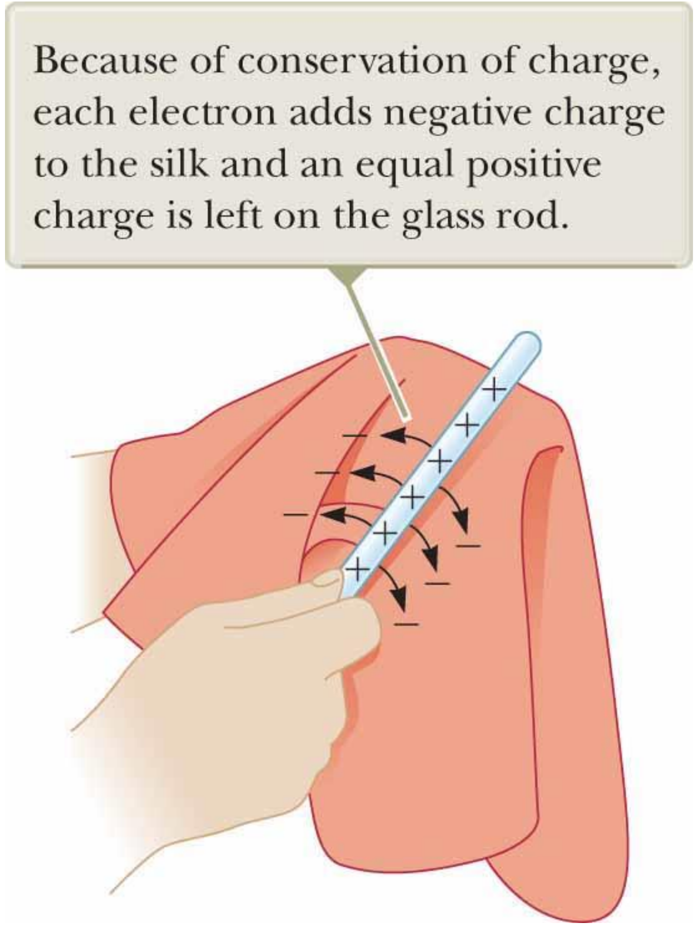
Properties of Electric Charges

Charges of opposite signs attract one another and charges with same sign repel one another .



Conservation of Charge

Electric charge is always conserved in isolated system



Charge is Quantized

Robert A. Millikan

Electric charge always occurs as integral multiples of fundamental amount of charge *e*:

$$q = \pm Ne$$

Electric charge *q* is quantized

$$e = 1.60218 \times 10^{-19} \text{ C}$$

$$1 \text{ C} = 6.2415 \times 10^{18} \; e$$

Conductors and Insulators

Electrical conductors are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material; electrical insulators are materials in which most electrons are bound to atoms and cannot move freely through the material.

$n_{e\ \mathrm{metal}} \sim 10^{22}\ \mathrm{cm}^{-3}$

$n_{e\ \mathrm{insulator}} \sim 10^{8}\ \mathrm{cm}^{-3}$

Semiconductors

silicon

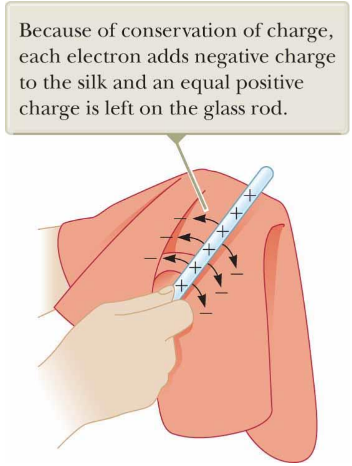
germanium

$$n_{e\ \mathrm{semiconductor}} \sim 10^{21} \div 10^{15} \div 10^{8}\ \mathrm{cm}^{-3}$$

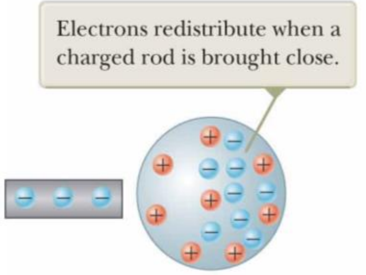
Charging Objects

Objects could be charged by:

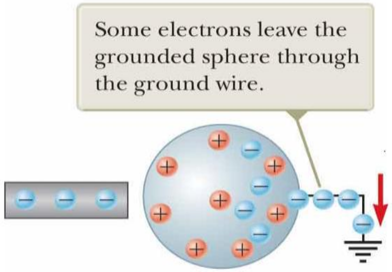
- rubbing (charge splitting up),



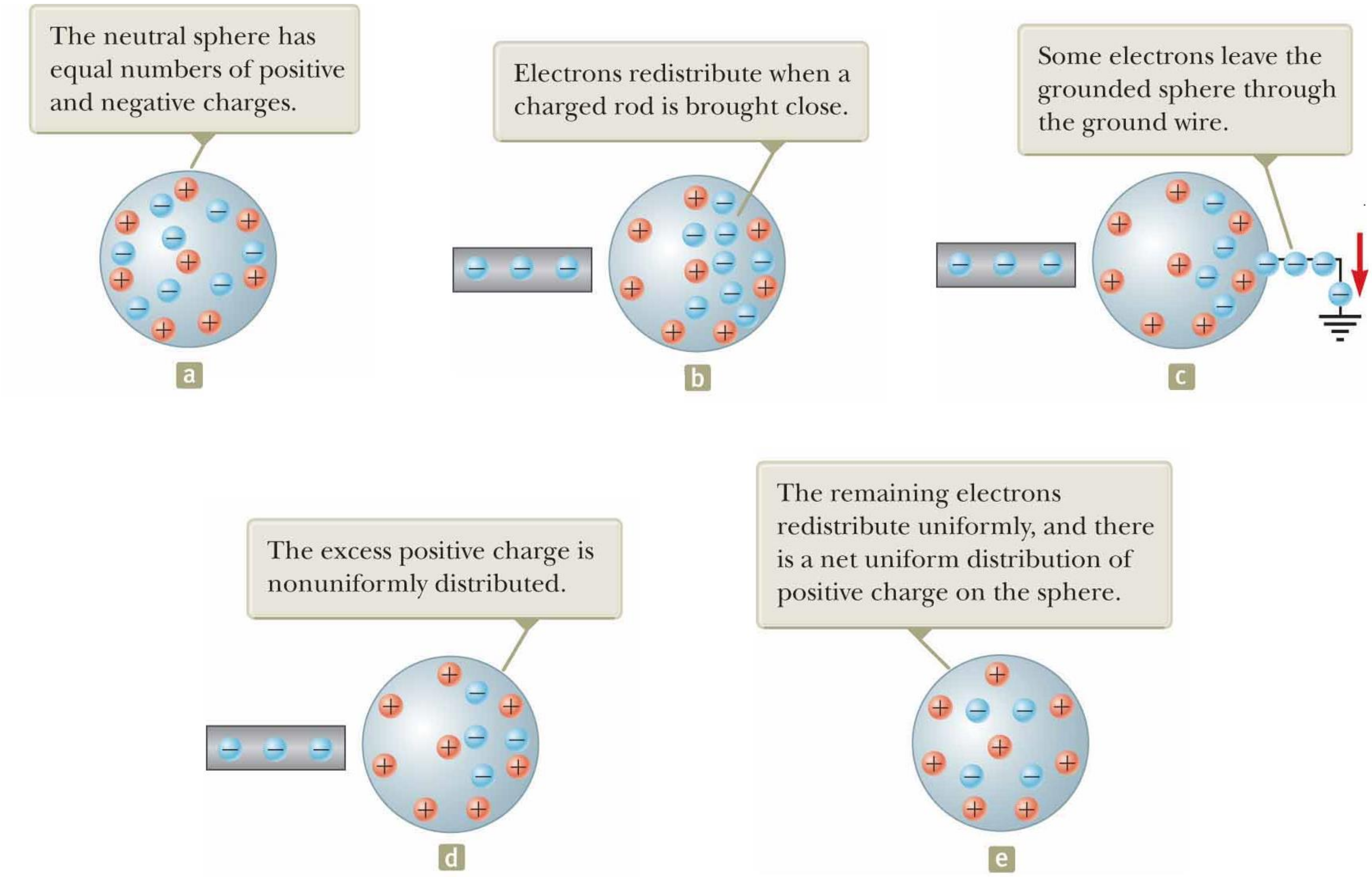
- induction (charge separation),



- touching (charge flow).



Charging Objects by Induction



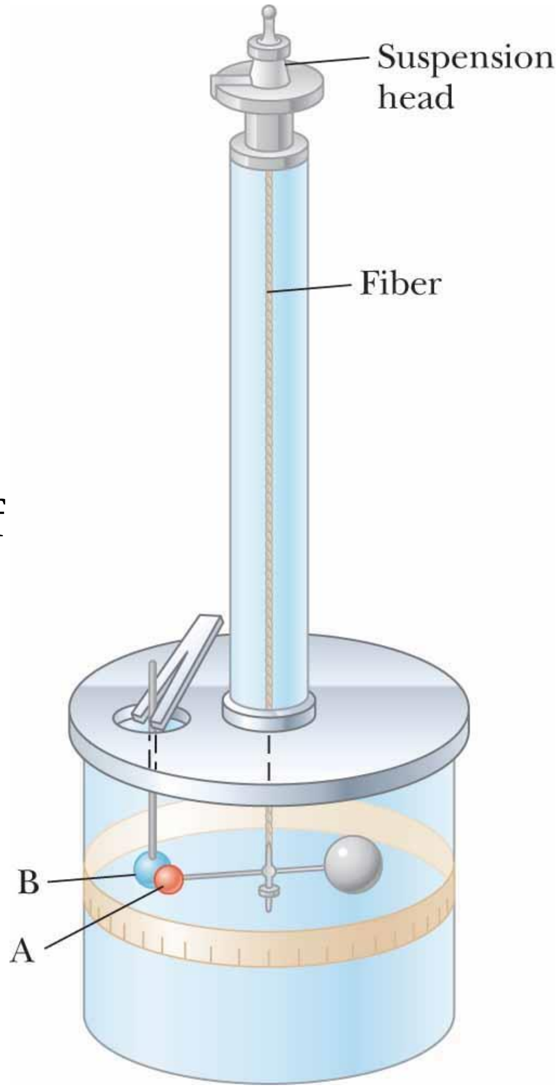
Electric Forces Between Charged Objects

Charles Coulomb : torsion balance

Electric force between charged spheres A and B causes spheres to either attract or repel each other.

Resulting motion causes suspended fiber to twist.

Measurement of angle provides quantitative measure of electric force of attraction or repulsion.



Coulomb's Law (1784)

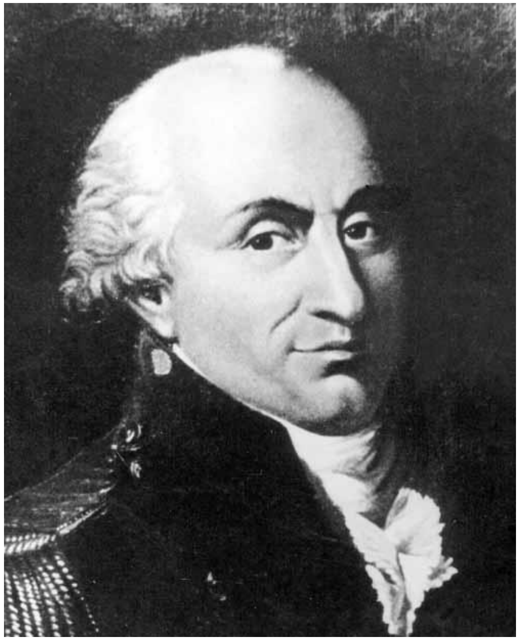
The magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them:

$$\boxed{F_e = k_e \frac{|q_1||q_2|}{r^2}}$$

$$k_e = 8.9876 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

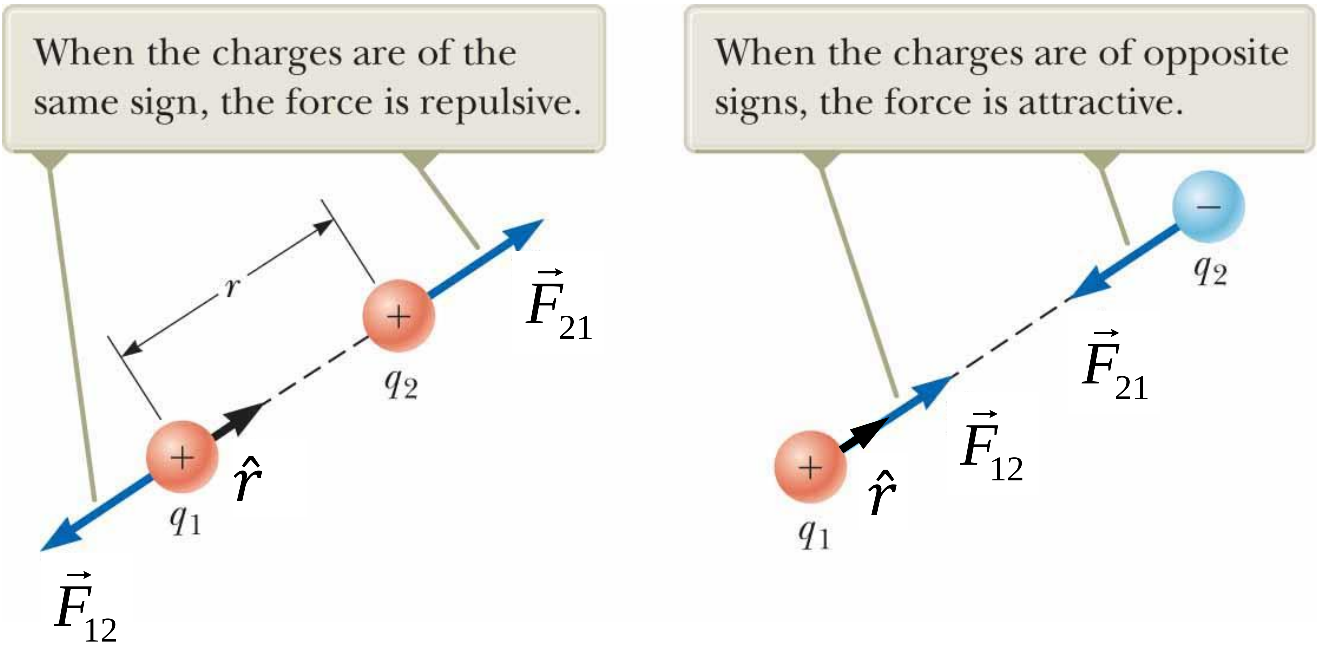
$$k_e = \frac{1}{4\pi\varepsilon_0}$$

Charles Coulomb



$\varepsilon_0 = 8.8542 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$ – the absolute permittivity of free space

Coulomb's Law in Vector Form



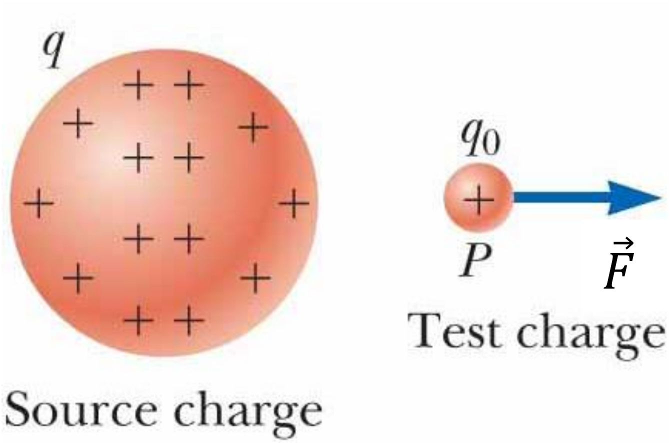
$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

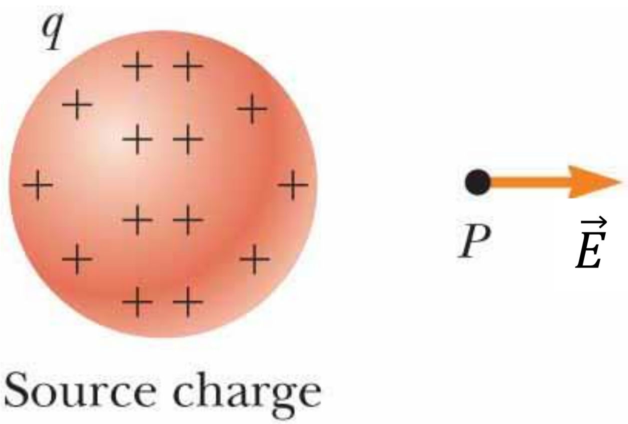
$$\hat{r} = \frac{\vec{r}}{r}$$

unit vector directed from q_1 toward q_2

Electric Field

Every charged particle in the universe creates an electric field in the space surrounding it, described by electric field vector $\vec{E}$ defined as electric force $\vec{F}$ acting on positive test charge q_0 placed at that point, divided by test charge.



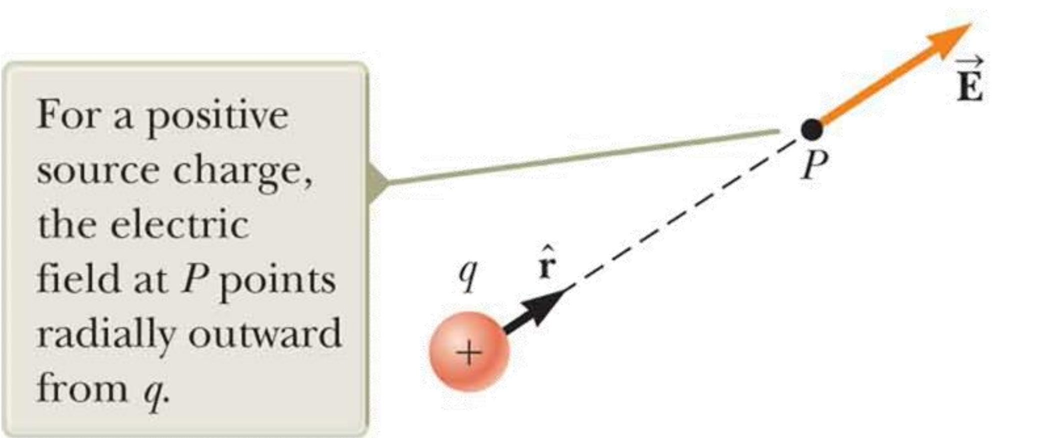


$$\vec{F} = q_0\vec{E}$$

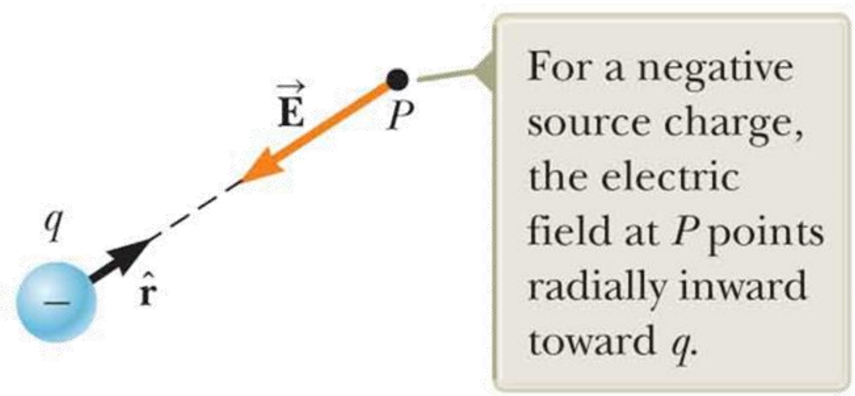
$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$

Electric Field

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r} \qquad \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$



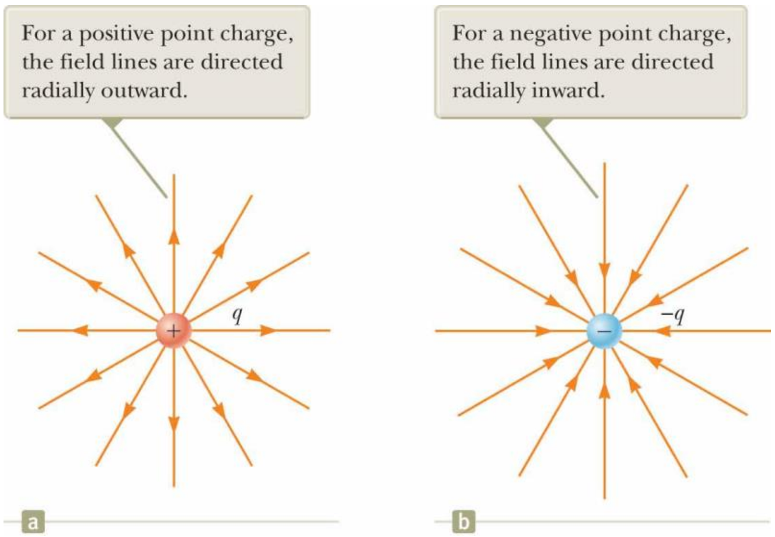
Electric field value is proportional to the source charge and is inversely proportional to the square of the distance from source charge (field weaken with distance.).



Electric Field Lines

A convenient way of visualizing electric field patterns is to draw lines, called electric field lines and first introduced by Faraday, that are related to the electric field in a region of space in the following manner:

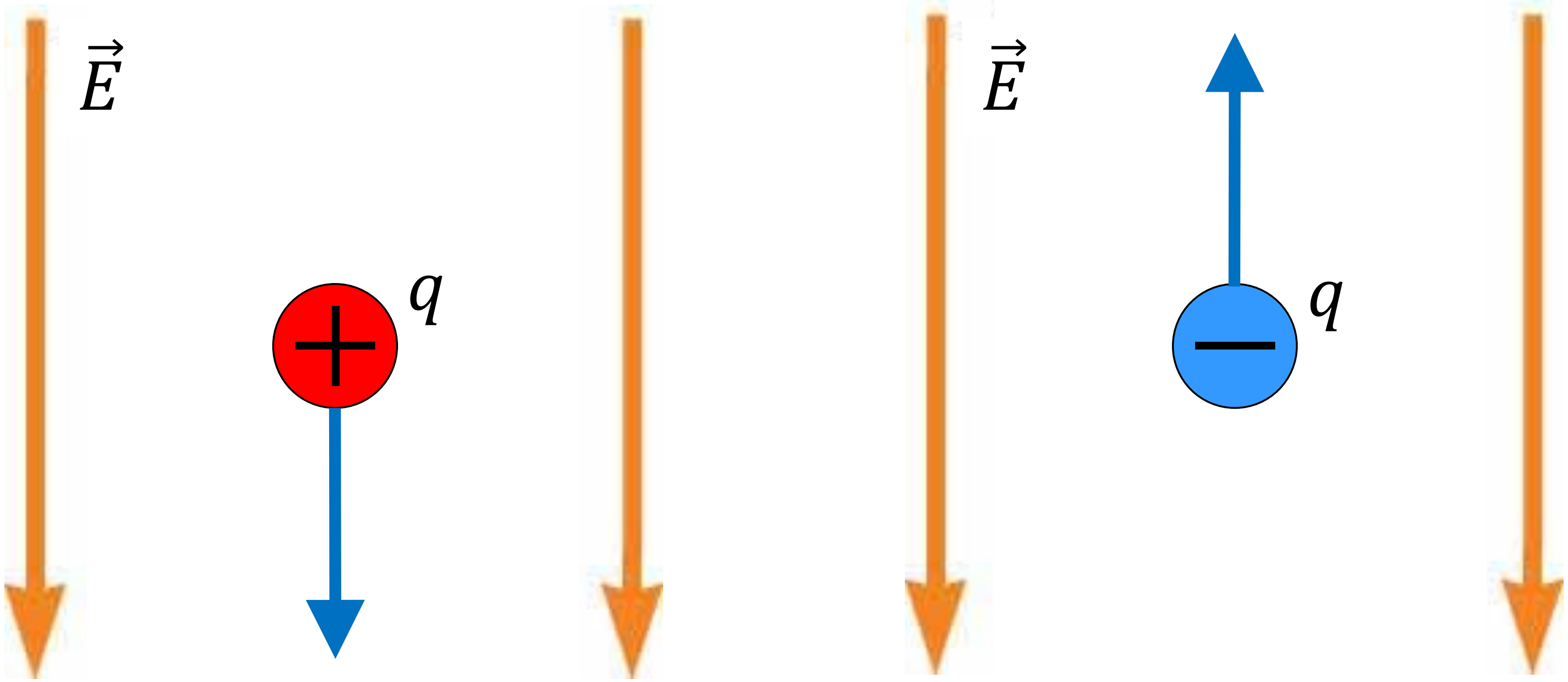
- The electric field vector $\vec{E}$ is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector. The direction of the line is that of the force on a positive charge placed in the field according to the particle in a field model.



- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Therefore, the field lines are close together where the electric field is strong and far apart where the field is weak.

Particle in an Electric Field

$$\vec{F} = q\vec{E}$$



Superposition principle

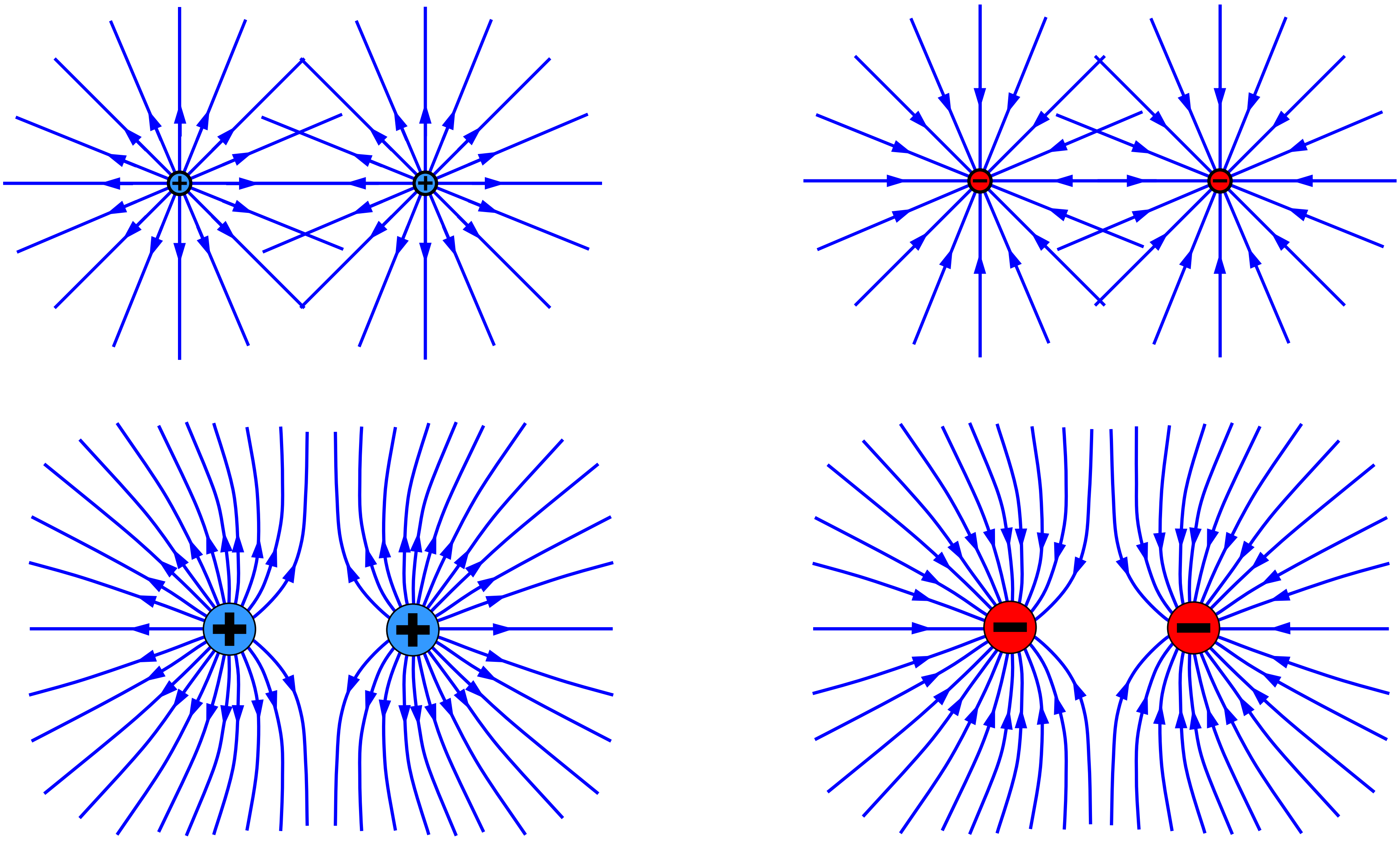
The **principle of superposition** states that:

1) every charge in space creates an electric field at point **independent** of the presence of other charges in that medium.

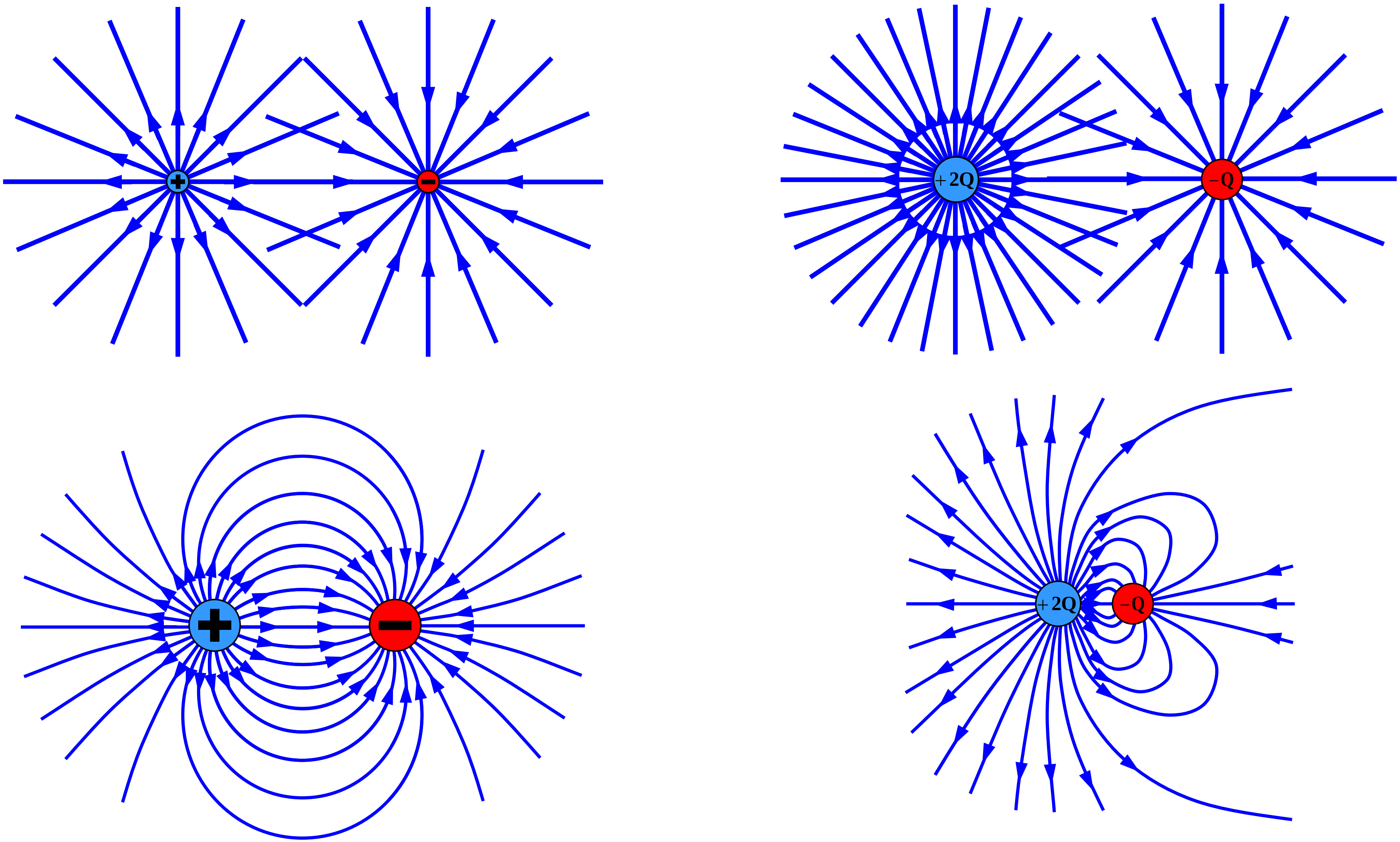
2) The **net electric field** is a vector sum of the electric field due to individual charges.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

Electric Fields Superposition



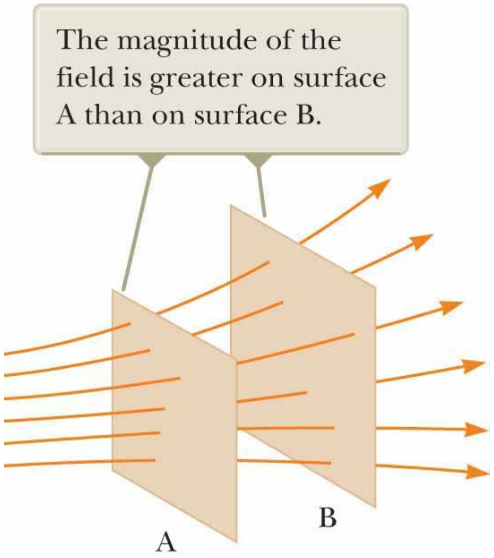
Electric Fields Superposition

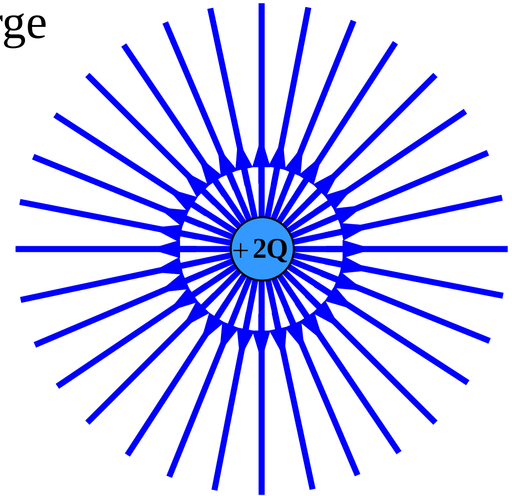


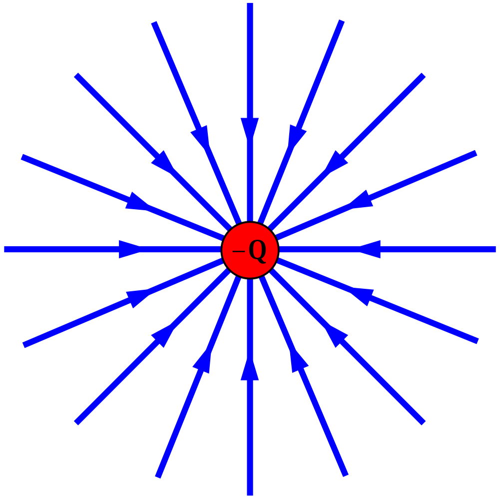
Electric Field Lines

Electric field lines:

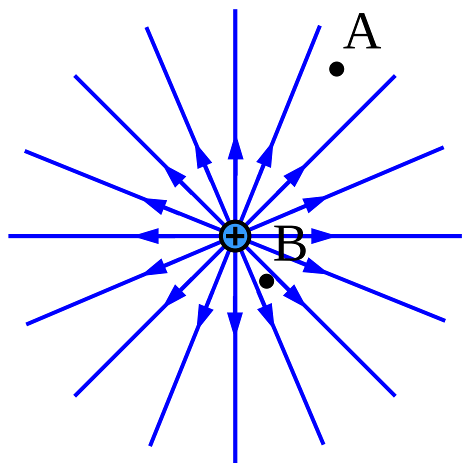
- Visualizing electric field in pictorial representation
- Electric field vector $\vec{E}$ is tangent to electric field line at each point
- Direction of line – that of force on positive charge
- Number of lines per unit area through surface perpendicular to lines is proportional to magnitude of electric field in region
- Number of field lines starting from or ending on any object is proportional to object charge





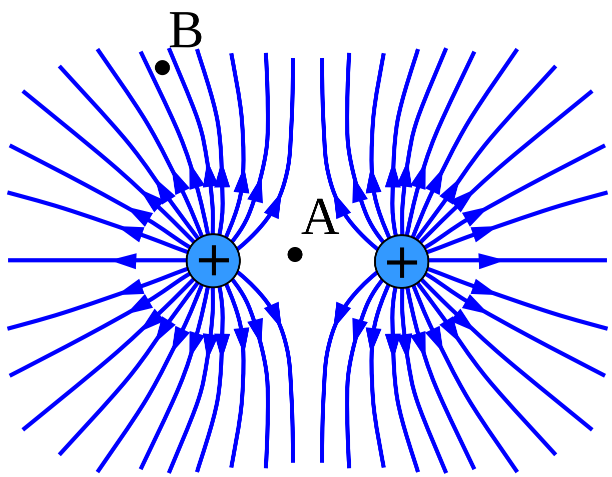


Electric Field Lines

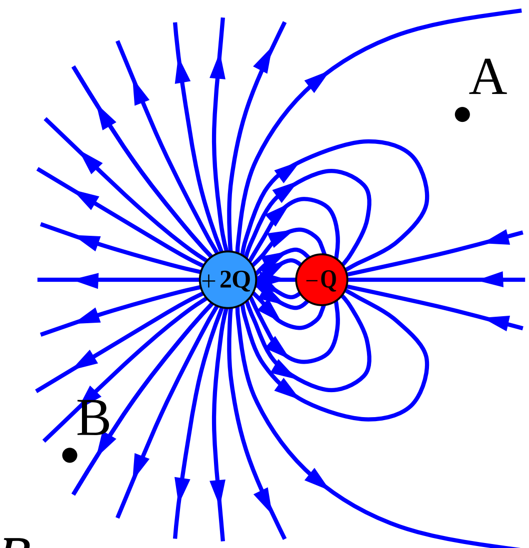


$E_A < E_B$

$E_A < E_B$



$$\frac{N_2}{N_1} = \left|\frac{Q_2}{Q_1}\right|$$



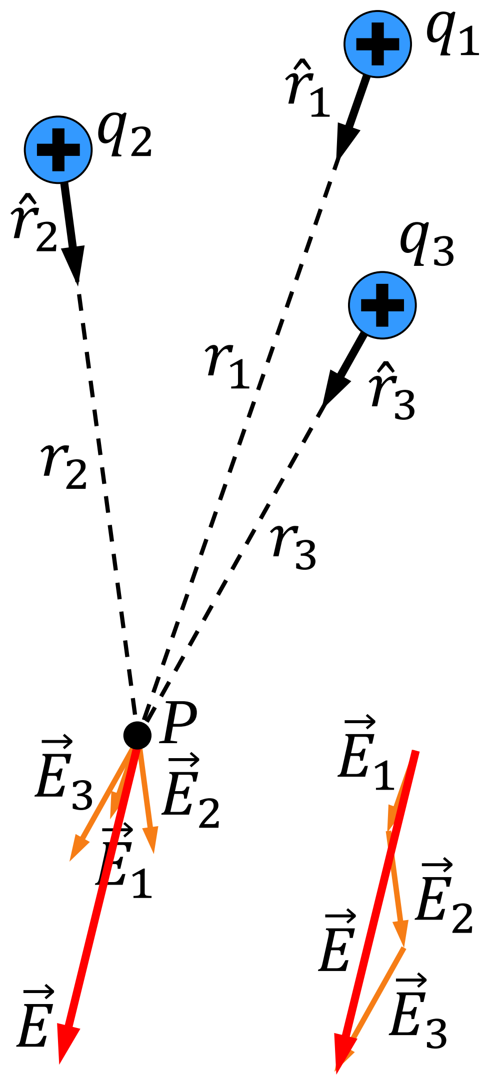
$E_A < E_B$

$$\frac{N_2}{N_1} = \left|\frac{Q_2}{Q_1}\right|$$

Electric Field of a Point Charge Distribution

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

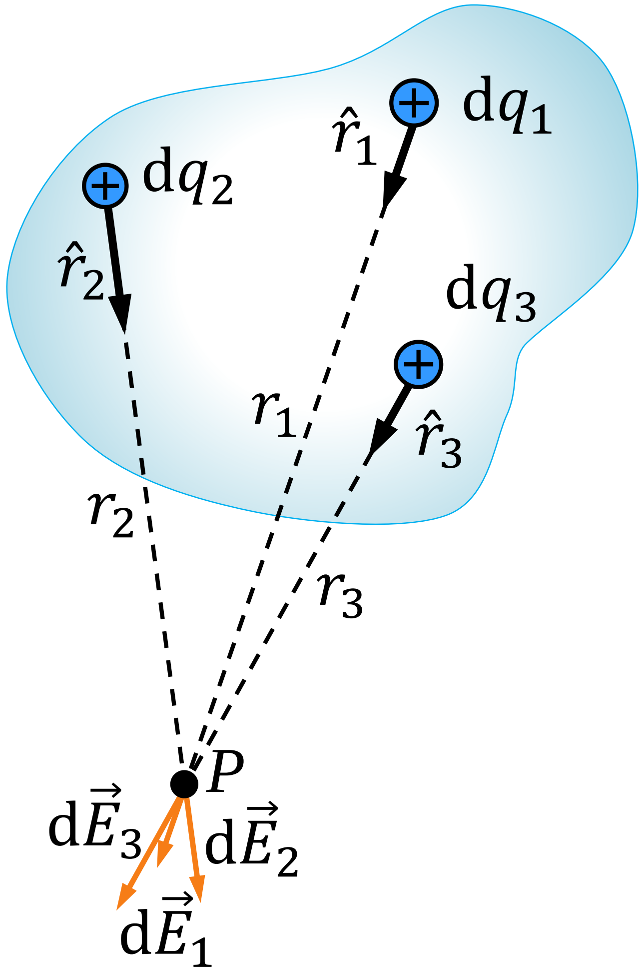


Electric Field of a Continuous Charge Distribution

$$d\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{dq_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \sum_i d\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{dq_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_Q \frac{dq}{r^2}\hat{r}$$



Charge Density

Charge density:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_Q \frac{\mathrm{d}q}{r^2}\hat{r}$$

1. volume charge density: $\rho \equiv \dfrac{Q}{V}$ $\quad dq = \rho\, dV \quad$ $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_V \frac{\rho}{r^2}\hat{r}\mathrm{d}V$

2. surface charge density: $\sigma \equiv \dfrac{Q}{A}$ $\quad dq = \sigma\, dA \quad$ $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_A \frac{\rho}{r^2}\hat{r}\mathrm{d}A$

3. linear charge density: $\lambda \equiv \dfrac{Q}{\ell}$ $\quad dq = \lambda\, d\ell \quad$ $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_A \frac{\lambda}{r^2}\hat{r}\mathrm{d}\ell$

Electric Field Due to a Charged Rod

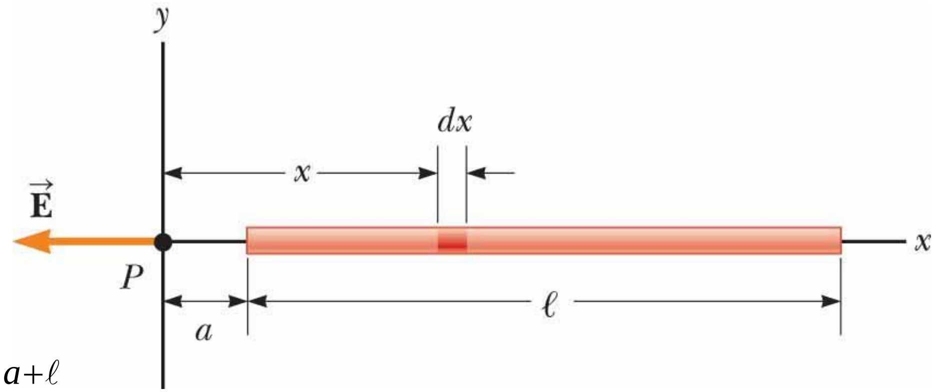
A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.

$$\mathrm{d}E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x^2}\mathrm{d}x$$

$$E = \int_a^{a+\ell}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x^2}\mathrm{d}x$$

$$E = \frac{1}{4\pi\varepsilon_0}\lambda\int_a^{a+\ell}\frac{1}{x^2}\mathrm{d}x = \frac{1}{4\pi\varepsilon_0}\lambda\left[-\frac{1}{x}\right]_a^{a+\ell}$$

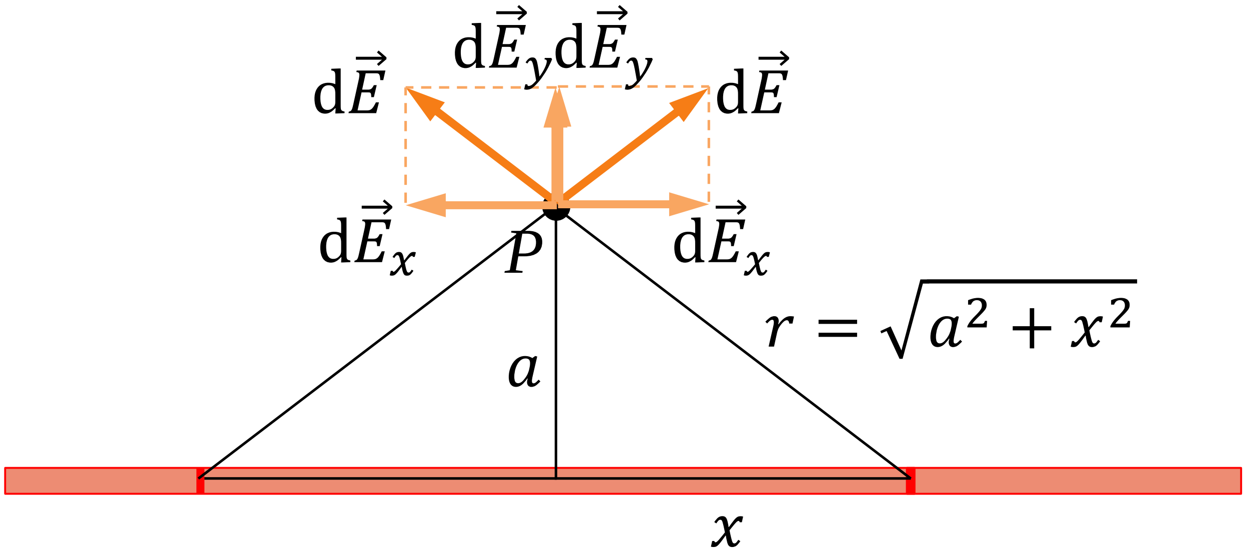
$$E = \frac{1}{4\pi\varepsilon_0}\lambda\left(-\frac{1}{a+\ell}-\left(-\frac{1}{a}\right)\right) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\ell}{a(a+\ell)}$$



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a(a+\ell)}$$

Electric Field Due to a Charged Rod

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

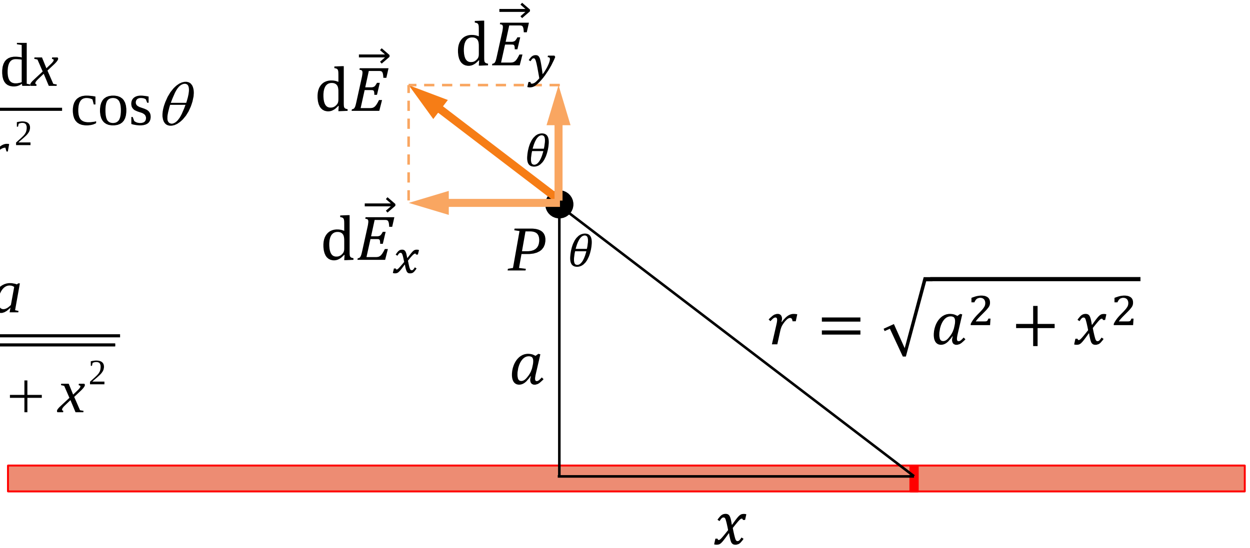


Because of the symmetry of the situation, the electric field $\mathrm{d}\vec{E}_x$ contributions from two segments on opposite sides of the rod cancel. That is true for successive pairs of segments around the rod, so we can ignore that component of the field and focus solely on the components $\mathrm{d}\vec{E}_y$, which simply add.

Electric Field Due to a Charged Rod

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

$$\mathrm{d}E_y = \mathrm{d}E\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \mathrm{d}x}{r^2}\cos\theta$$



$$\mathrm{d}E_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \mathrm{d}x}{a^2 + x^2}\frac{a}{\sqrt{a^2 + x^2}}$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{\lambda a}{\left(a^2 + x^2\right)^{\frac{3}{2}}}\mathrm{d}x = \frac{1}{4\pi\varepsilon_0}\lambda a\left[\frac{x}{a^2\sqrt{a^2 + x^2}}\right]_{-\infty}^{+\infty} = \frac{1}{4\pi\varepsilon_0}\lambda a\frac{2}{a^2}$$

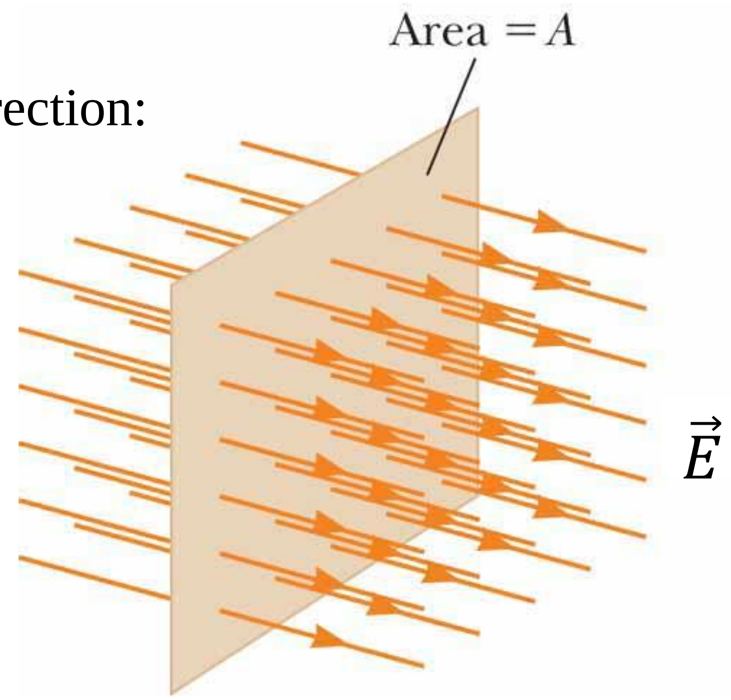
$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

Electric Flux

Electric flux is the measure of flow of the electric field through a given area.

Consider electric field uniform in magnitude and direction:

$$\Phi_E = EA$$



$\vec{E}$

Electric Flux

$$A = \ell w$$

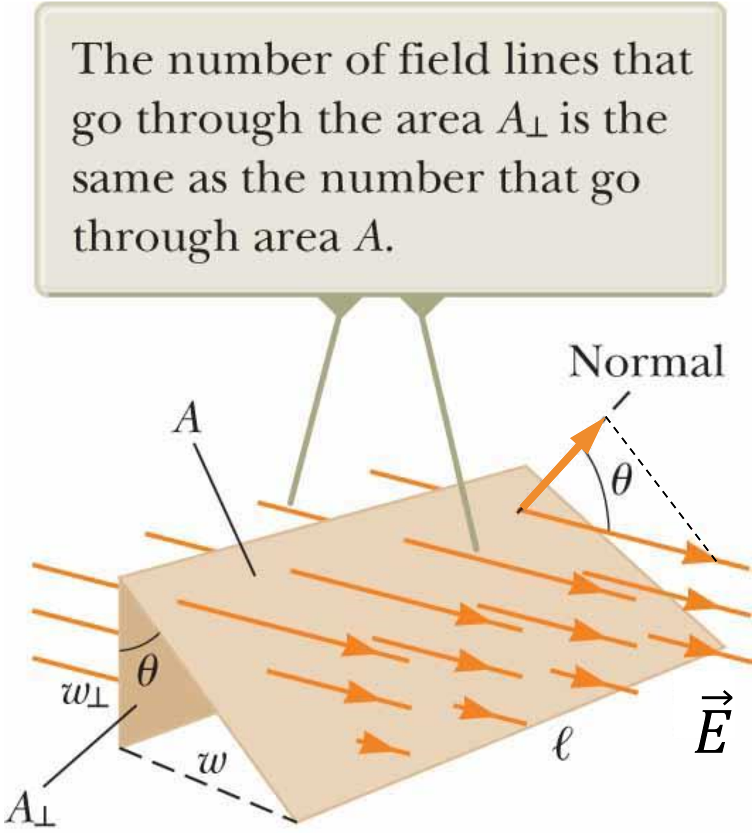
$$w_\perp = w\cos\theta$$

$$A_\perp = \ell w_\perp = \ell w\cos\theta$$

$$A_\perp = A\cos\theta$$

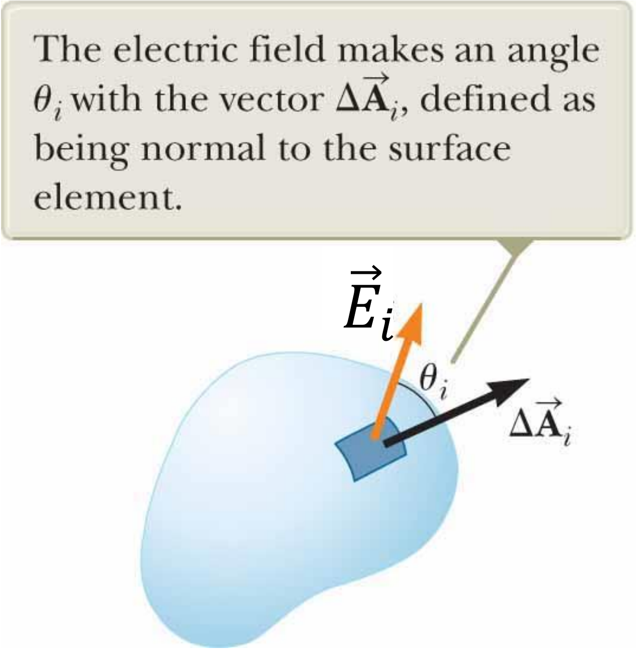
$$\Phi_E = EA_\perp = EA\cos\theta$$

$$\Phi_E = \left(E\cos\theta\right)A = E_n A$$



Electric Flux

$$\Phi_{E,i} = E_i \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$



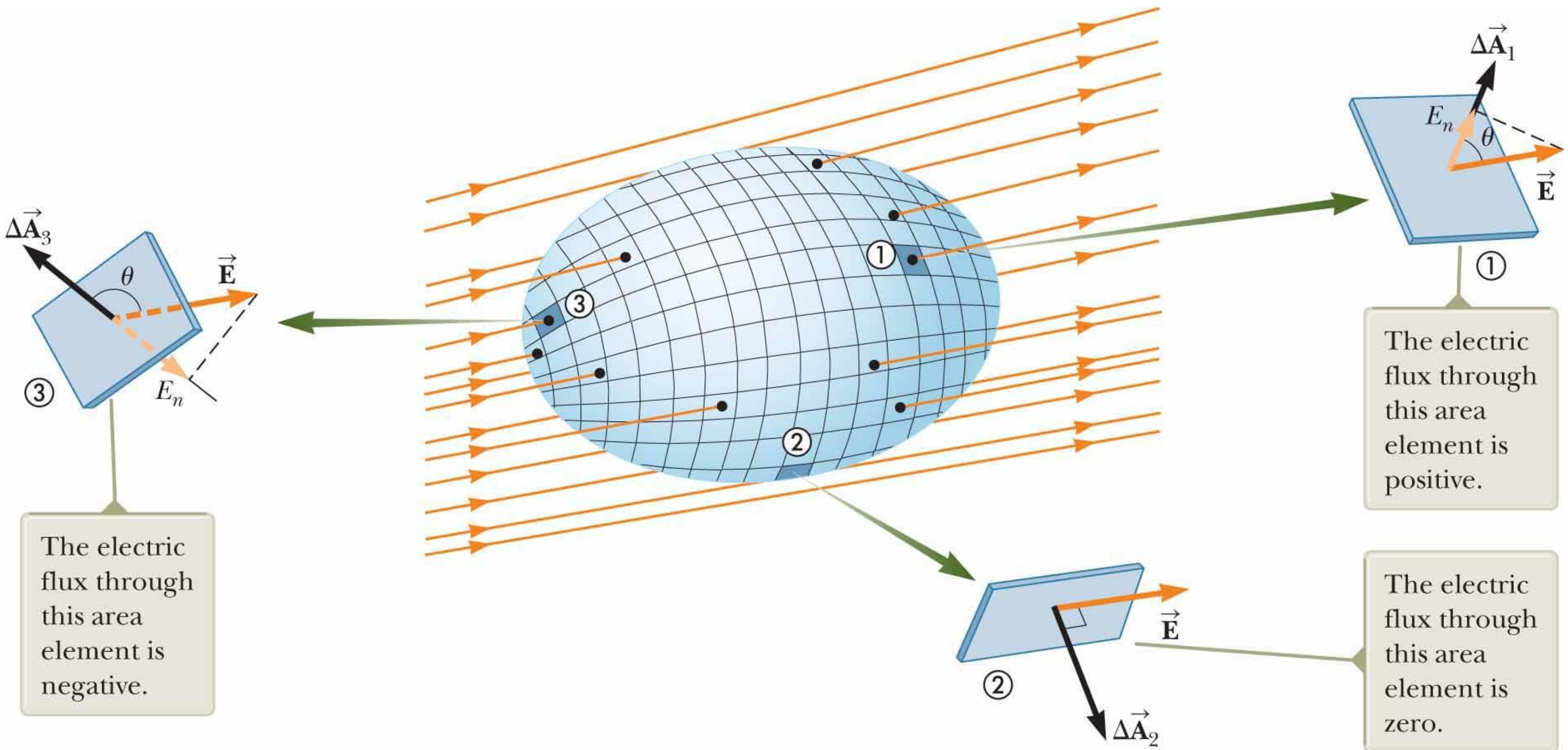
The total electric flux over a surface S is therefore given by the surface integral

$$\Phi_E \equiv \int_{\text{surface}} \vec{E} \cdot \mathrm{d}\vec{A}$$

where $\vec{E}$ is the electric field and $d\vec{A}$ is a differential area on the closed surface with an outward facing surface normal defining its direction.

Electric Flux

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = \oint E_n \cdot \mathrm{d}A$$



Gauss's Law

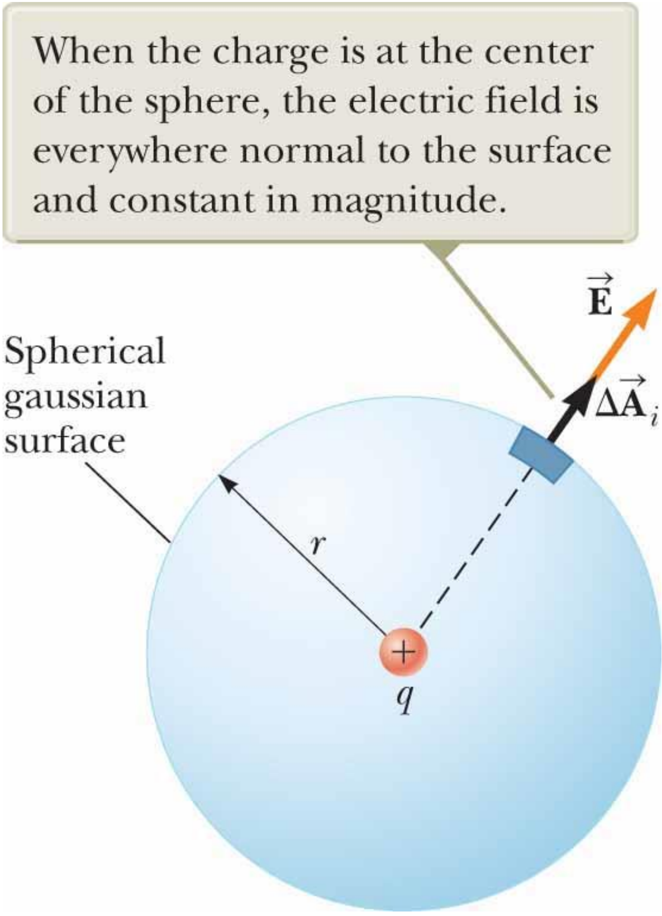
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}_i = E\Delta A_i$$

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E dA = E\oint dA$$

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left(4\pi r^2\right)$$

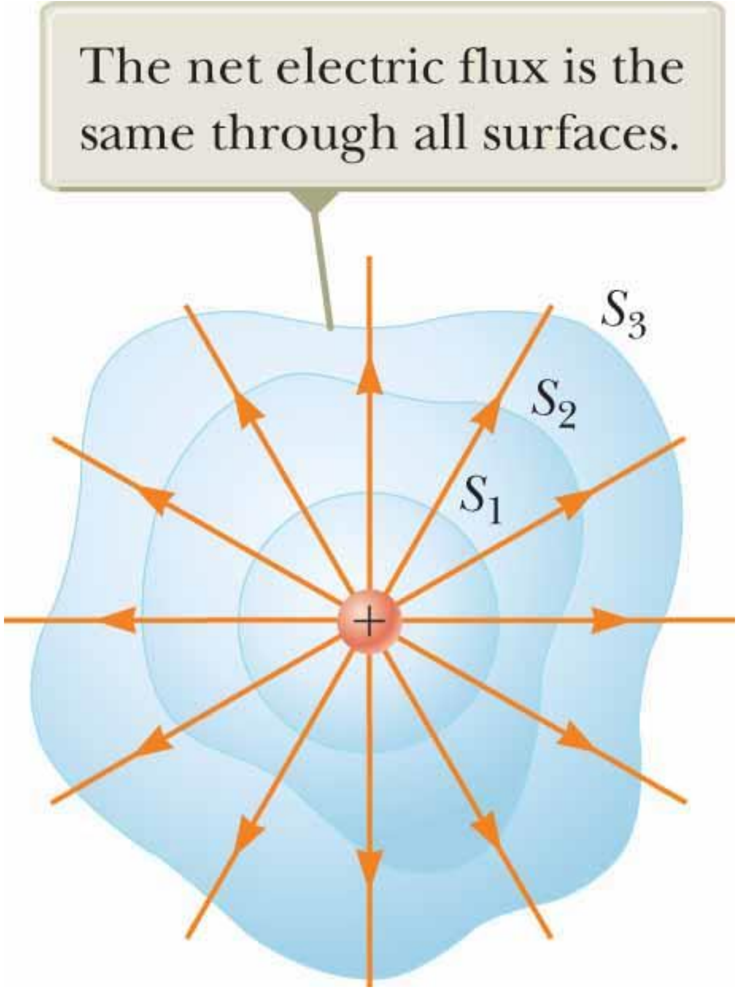
$$\Phi_E = \frac{q}{\varepsilon_0}$$



Gaussian Surface and Flux

Net flux through any closed surface surrounding a point charge q is given by q/ε_0 and is independent of the shape of that surface.

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$



A Spherically Symmetric Charge Distribution

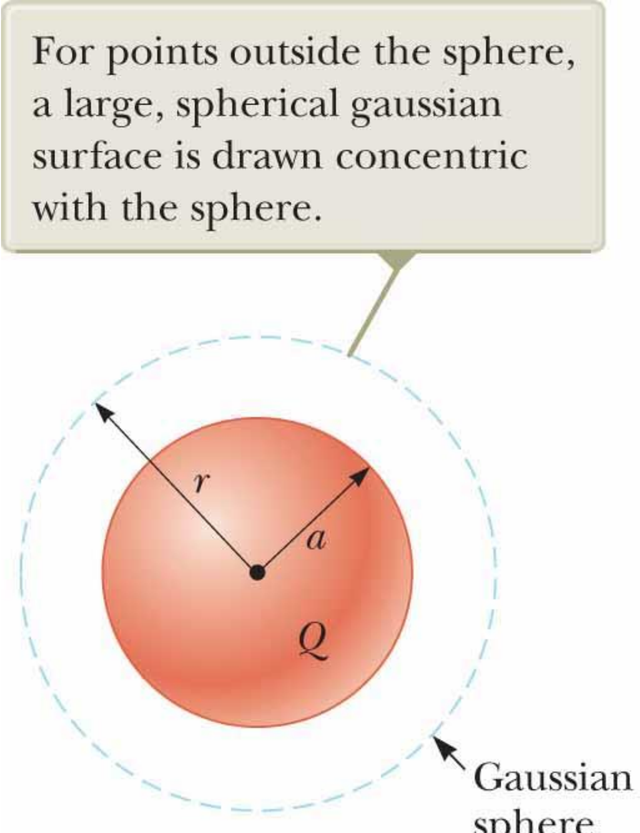
An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q.

(A) Calculate the magnitude of the electric field at a point outside the sphere.

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = \oint E\,\mathrm{d}A = \frac{Q}{\varepsilon_0}$$

$$\oint E\,dA = E\oint dA = E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (\text{for } r > a)$$



A Spherically Symmetric Charge Distribution

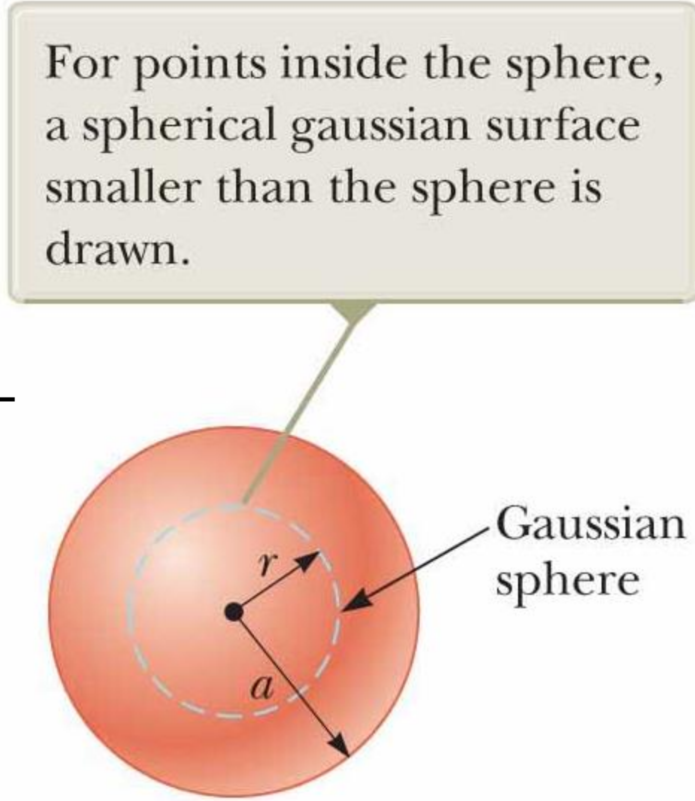
(B) Find the magnitude of the electric field at a point inside the sphere.

$$q_{\text{in}} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$\oint E\,\mathrm{d}A = E\oint \mathrm{d}A = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\varepsilon_0}$$

$$E = \frac{q_{\text{in}}}{4\pi r^2 \varepsilon_0} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{4\pi\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0} r$$

$$E = \frac{Q/\frac{4}{3}\pi a^3}{3\varepsilon_0} r = \frac{Q}{4\pi\varepsilon_0 a^3} r \quad \left(\text{for } r < a\right)$$

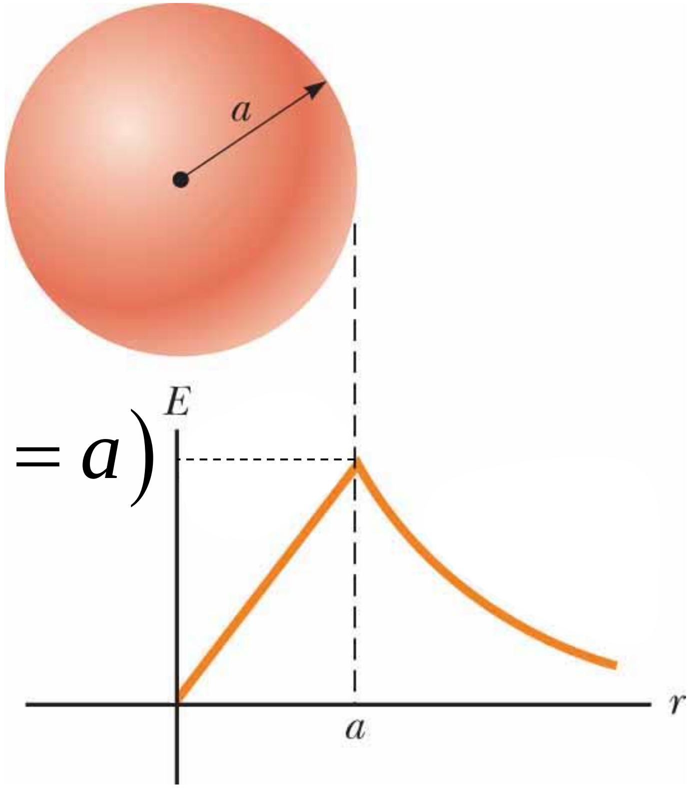


A Spherically Symmetric Charge Distribution

$$E = \frac{Q}{4\pi\varepsilon_0 a^3} r \quad (\text{for } r < a)$$

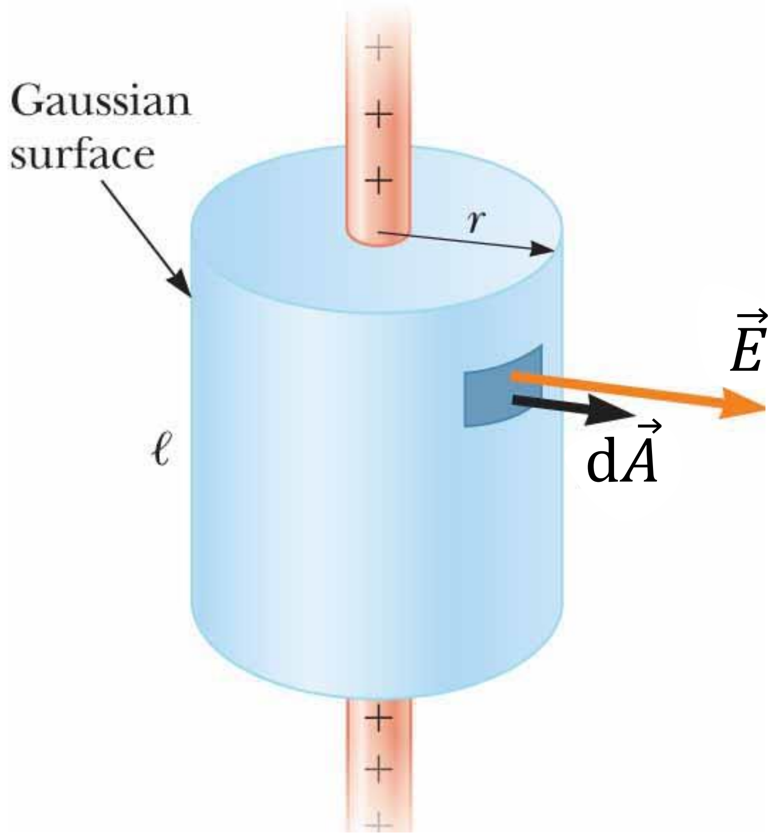
$$E = \frac{Q}{4\pi\varepsilon_0 a^2} \quad (\text{for } r = a)$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (\text{for } r > a)$$



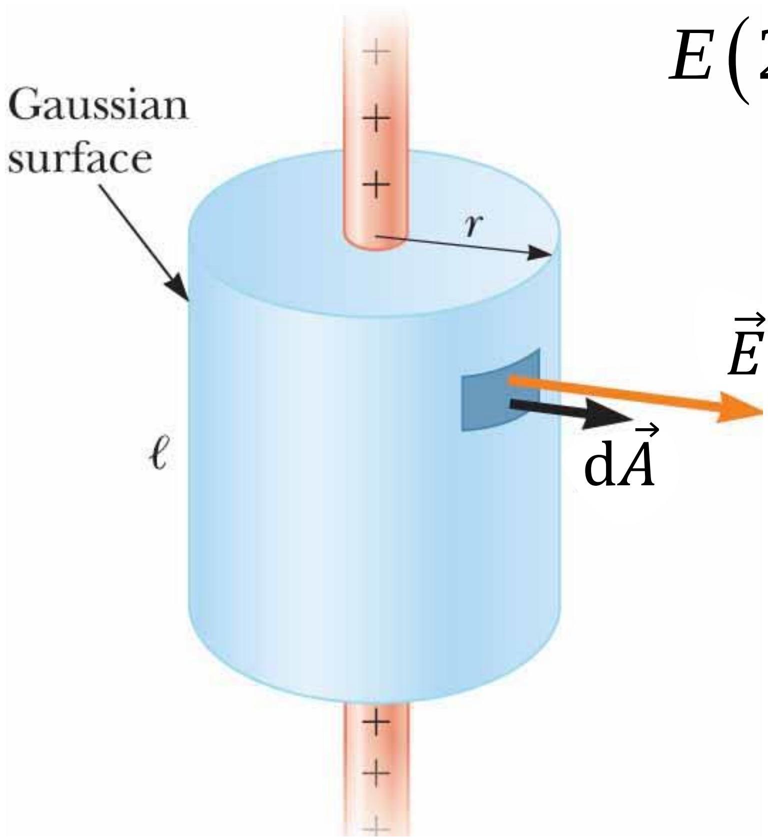
A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.



A Cylindrically Symmetric Charge Distribution

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = E \oint \mathrm{d}A = EA = \frac{q_{\mathrm{in}}}{\varepsilon_0} = \frac{\lambda \ell}{\varepsilon_0}$$



$$E(2\pi r \ell) = \frac{\lambda \ell}{\varepsilon_0}$$

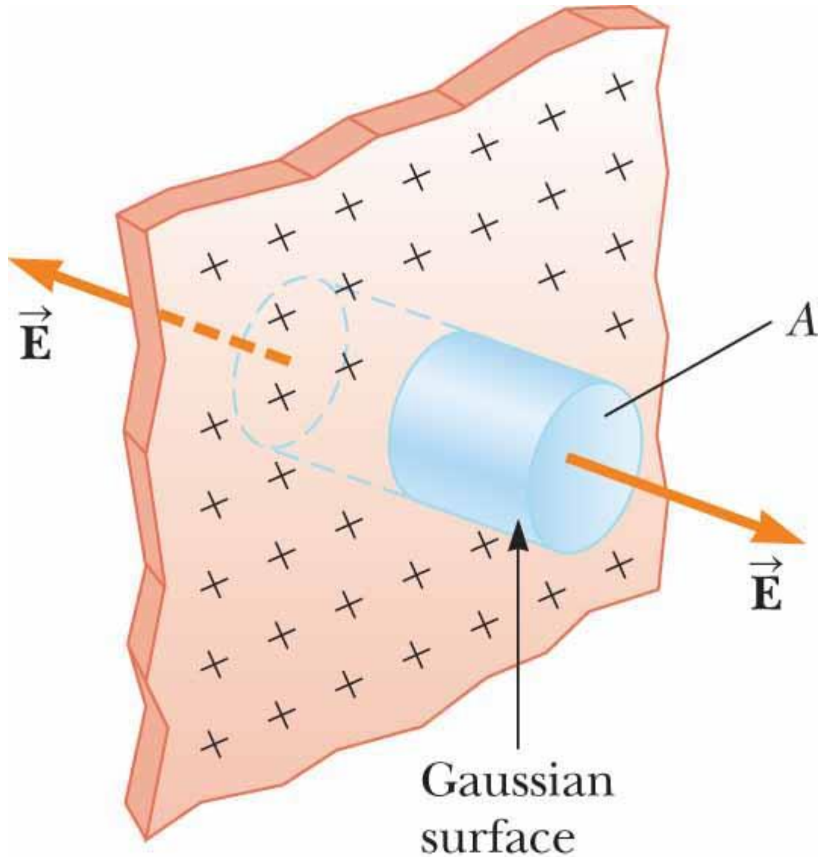
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

A Plane of Charge

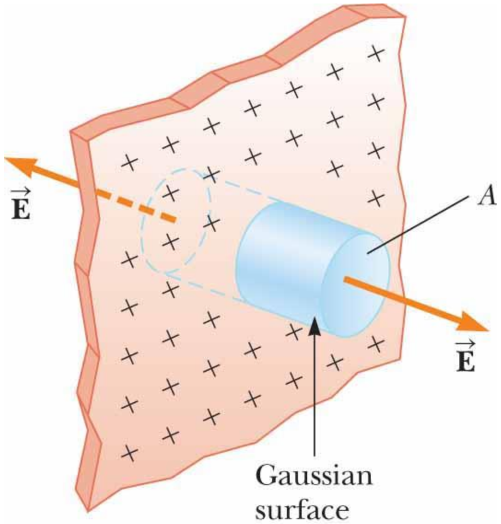
Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

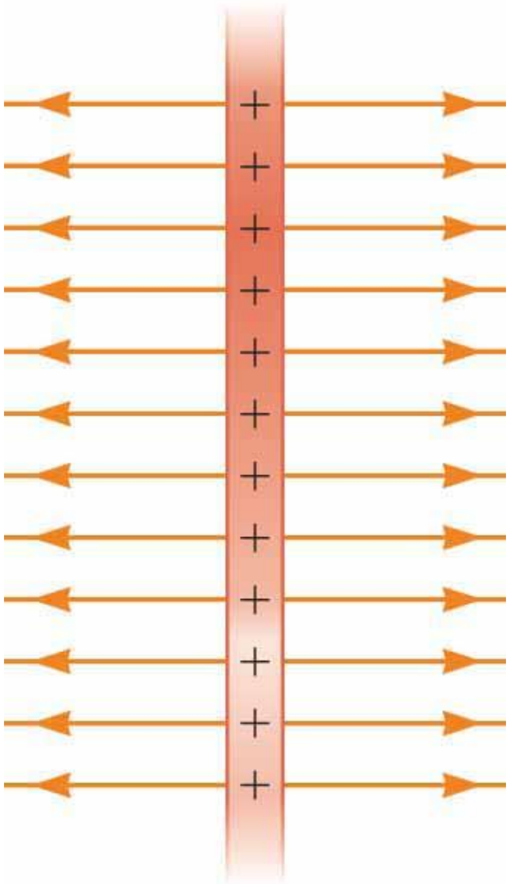
$$E = \frac{\sigma}{2\varepsilon_0}$$



A Plane of Charge



$$E = \frac{\sigma}{2\varepsilon_0}$$



Because the distance from each flat end of the cylinder to the plane does not appear in equation, $E = \sigma/2\varepsilon_0$ at *any* distance from the plane.

That is, the field is uniform everywhere – has the same value and direction.

Parallel Planes of Charge

The electric fields due to the two planes add in the region between the planes, resulting in a uniform field of magnitude σ/ε_0, and cancel elsewhere to give a field of zero. The figure shows the field lines for such a configuration. This method is a practical way to achieve uniform electric fields with finite-sized planes placed close to each other.